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# **Modelling Data – Better Approaches**

## **How to get useful information?**

Adrian R. Rennie



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# Monolayers – Simple Interpretation

Define  $g_s(Q_z)$  in terms of measured reflectivity and  $R_F(Q_z)$  (the Fresnel reflectivity for perfectly sharp interface):

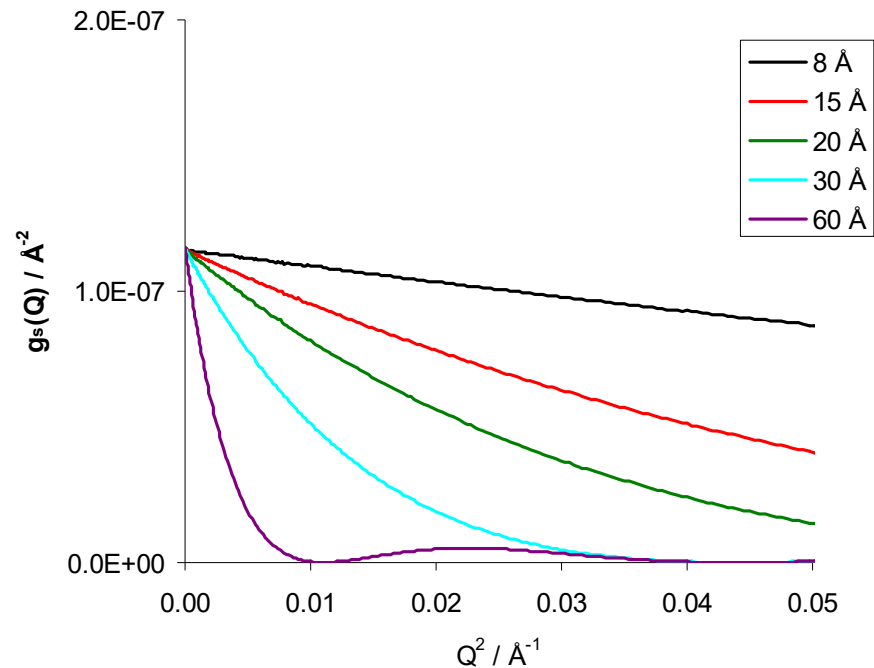
$$g_s(Q) = Q^2 (R - R_F) / (1 - R)$$

$$\ln g_s(Q) \approx -t^2 Q^2 / 12$$

$$\text{Roughly } \ln (Q^2 R) \approx -t^2 Q^2 / 12$$

Contrast match of two bulk phases

$$R_F(Q) = 0$$

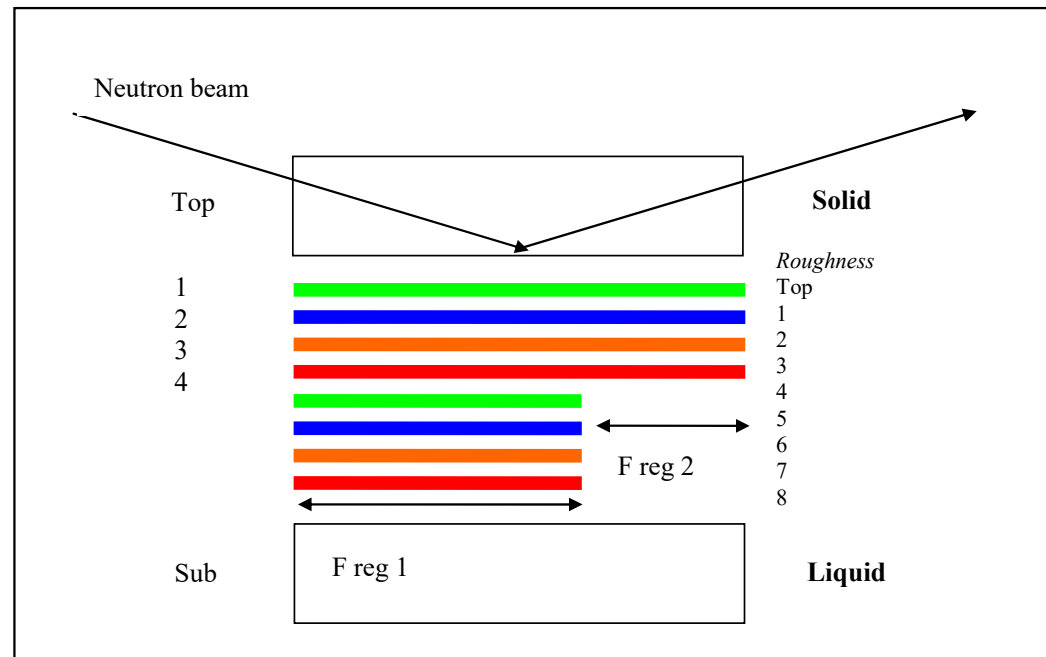




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# Real Interfaces are not just layers

Slab models are easy to calculate but people are not very interested in just thickness and scattering length density





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# Surface Excess and Area per Molecule

Volume per molecule:  
Scattering length:  $b_m$   
Scattering length density:  
 $\rho = b_m / V_m$

$V_m$

Thickness of layer:  
Scattering length density  
Area per molecule:

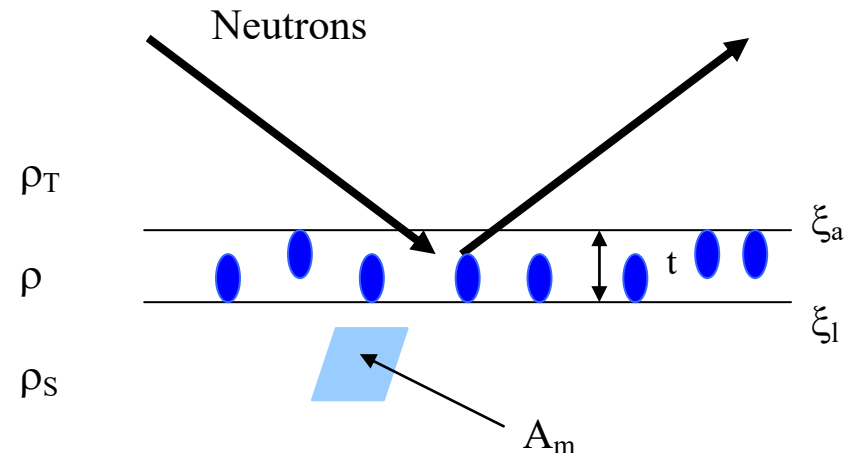
$t$

$\rho$

$A_m$

$V_m = t A_m$   
Scattering length density:  
 $\rho = (b_m / V_m) = b_m / (t A_m)$

Area per molecule:  $A_m = b_m / t \rho$





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# Adsorption of Surfactant

Surface active molecules

Amphiphilic

Bind to surface – how?

What are properties?

Hexadecyl trimethyl  
ammonium bromide



Tail

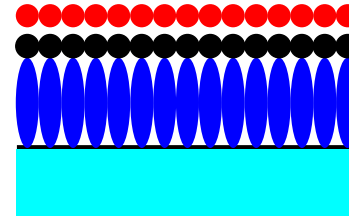
Head



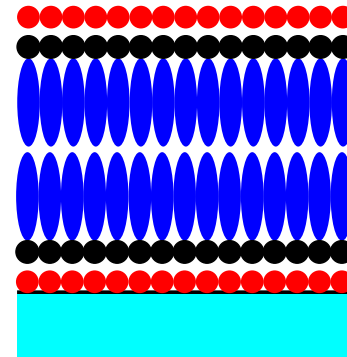
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# Some Possible Structures

- Monolayer



- Bilayer





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# Cationic Surfactant

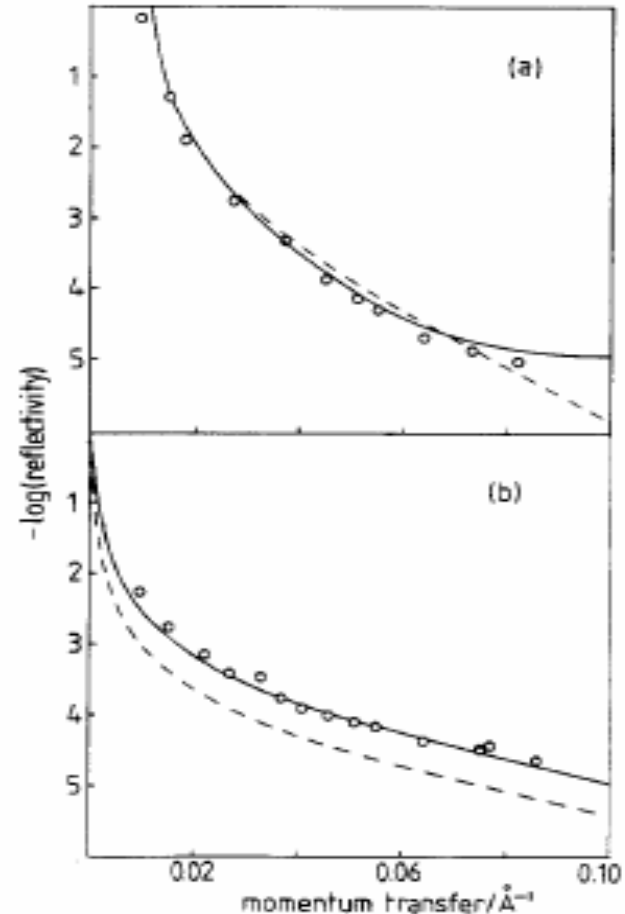
CTAB at 27° C on  
amorphous  $\text{SiO}_2$

(a)  $\text{D}_2\text{O}$  (b)  $\text{cmSiO}_2$   
at  $6 \times 10^{-4}$  M

## Models

Solid line – Bilayer

Dashed line - Monolayer





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# Cationic Surfactant

- **CTAB** 27 C on  $\text{SiO}_2$
- Label heads & tails

Head  $6 \pm 2 \text{ \AA}$

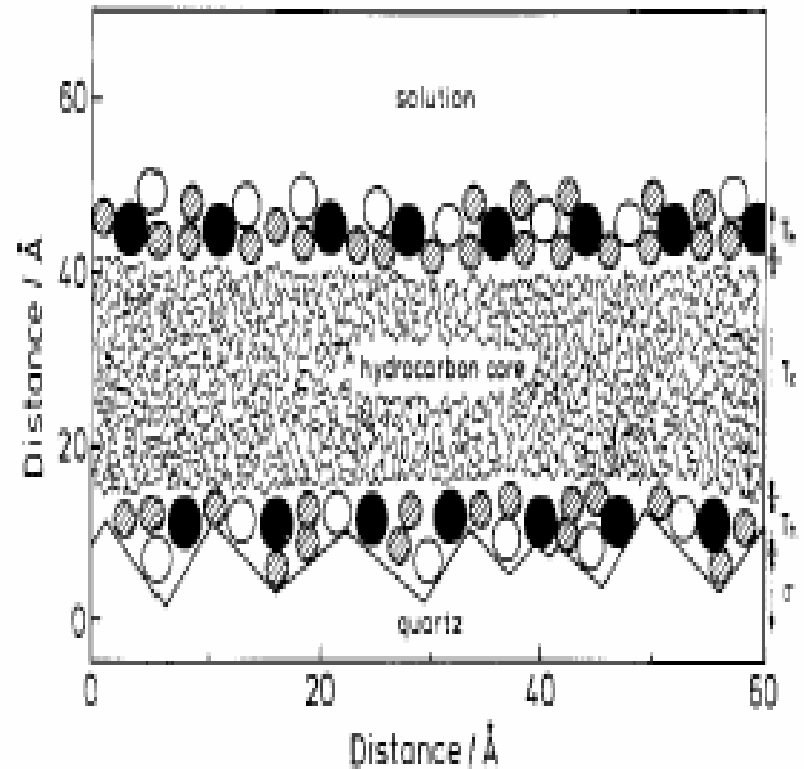
Tail  $28 \pm 4 \text{ \AA}$

Roughness  $\sim 8 \text{ \AA}$

Fractional Coverage

35% at  $3 \times 10^{-4} \text{ M}$

80% at  $6 \times 10^{-4} \text{ M}$



*Langmuir* **6**, 1031-1034 (1990).  
*J. Colloid Interf. Sci.* **162**, 304-310 (1994).

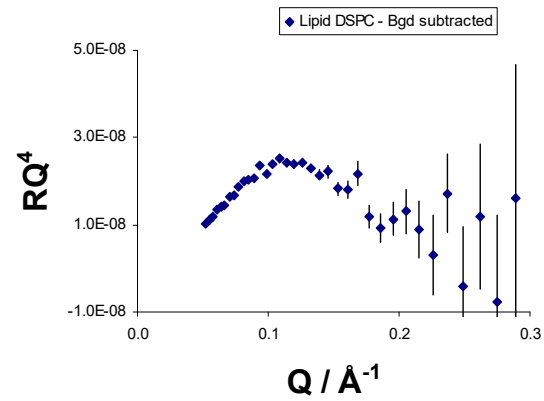
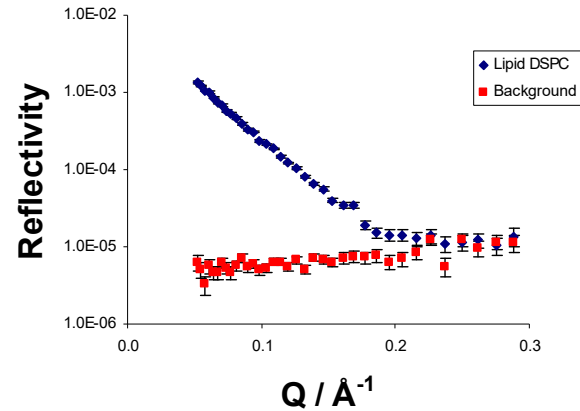
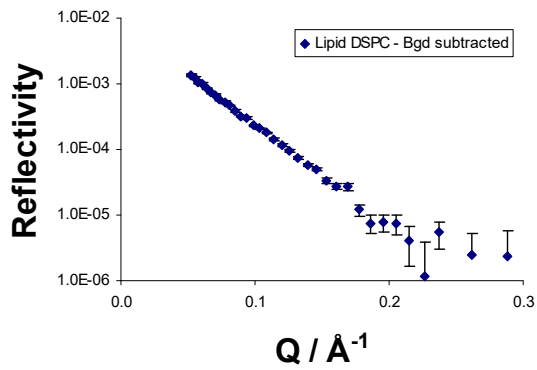




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# Plotting Data

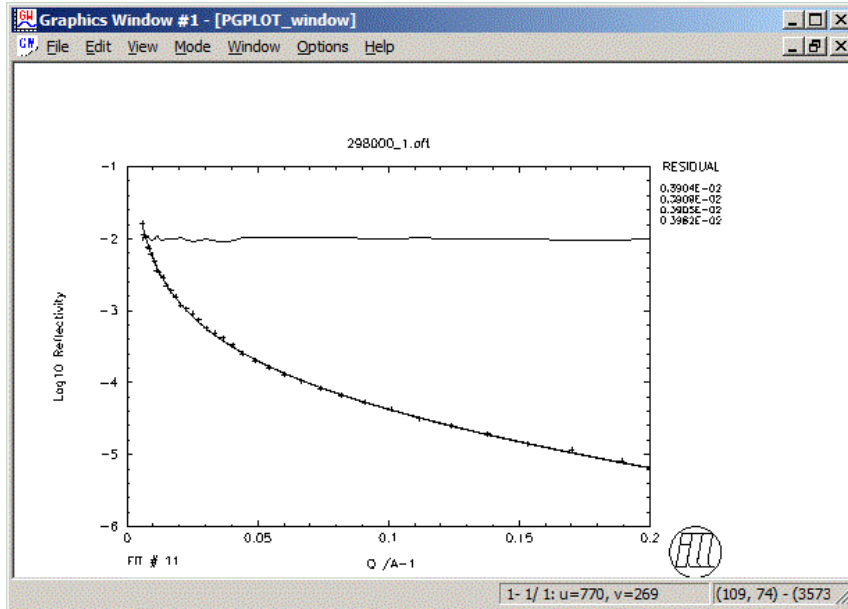
Different representation  
is helpful



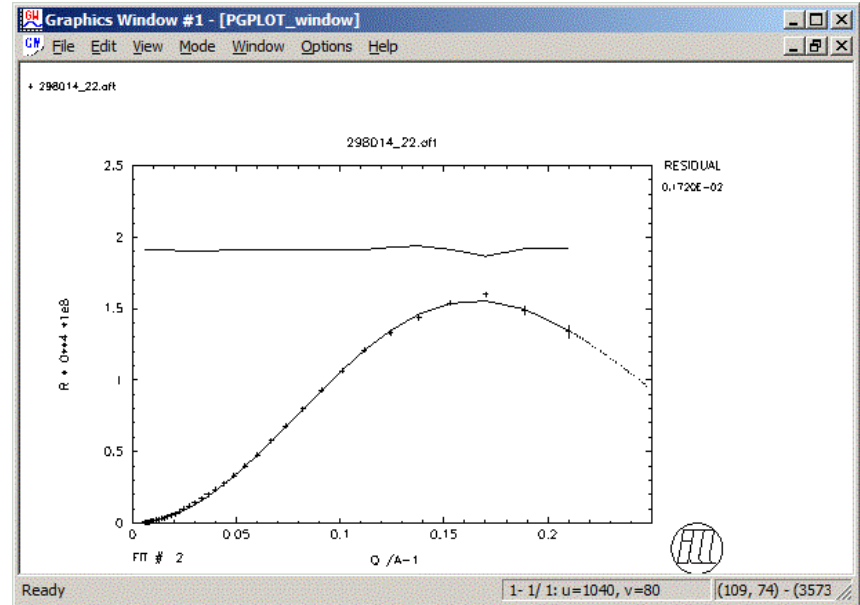


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# How to Look at Data?



Log<sub>10</sub> R vs Q

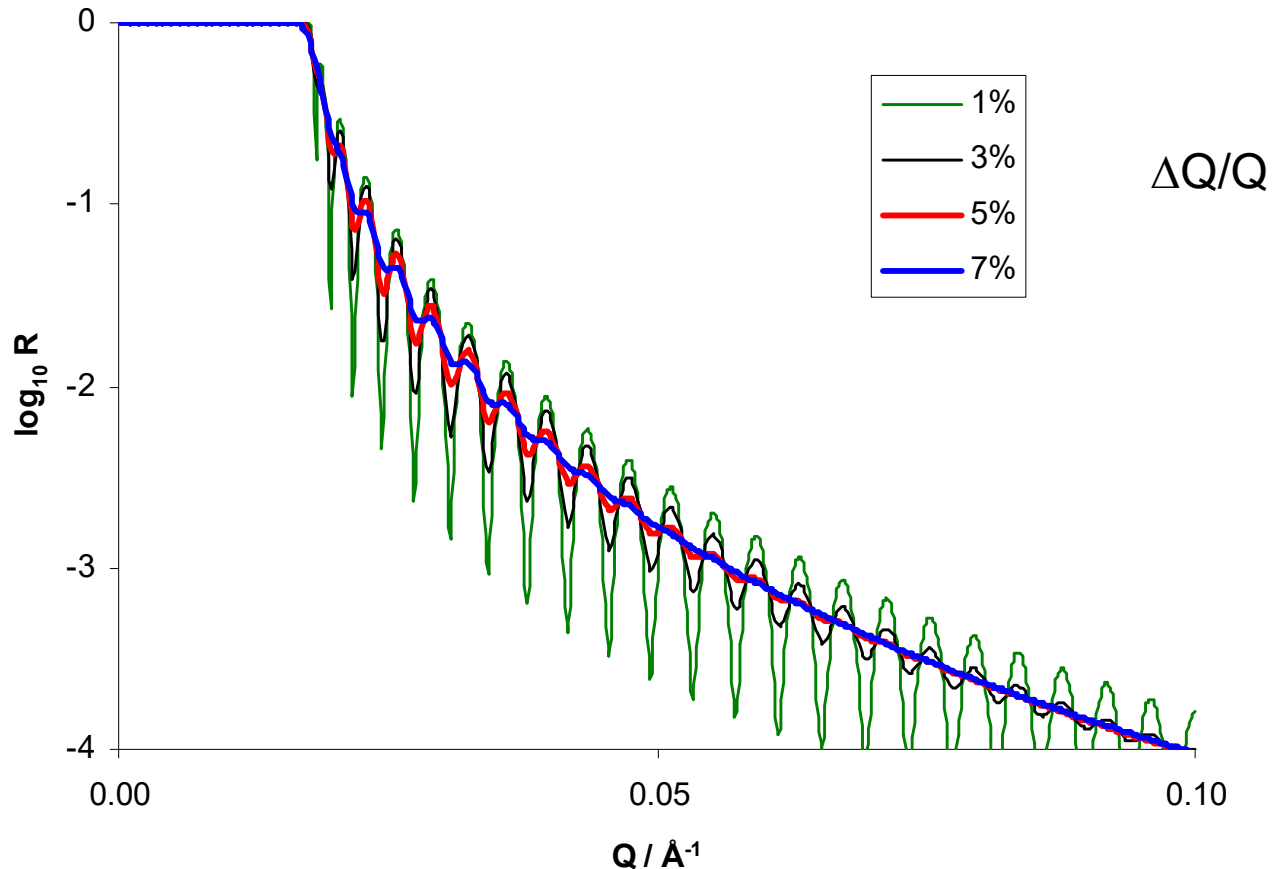


RQ<sup>4</sup> vs Q



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# Effects of Resolution



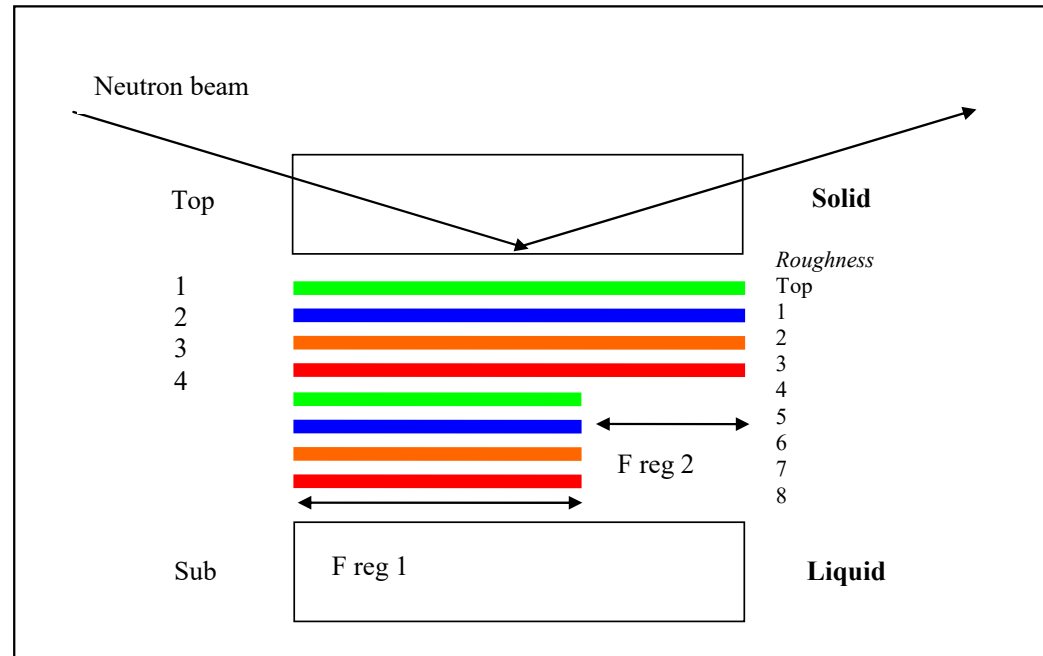
Silicon substrate: film thickness 1500  $\text{\AA}$   
scattering length density  $6.3 \times 10^{-6} \text{\AA}^{-2}$



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# Non-Uniform Surfaces

If you have patches of different layers at an interface do you average the density or average the reflectivity?



What is the coherence length of a neutron?



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# Describing Polymers

- Interdiffusion – is this roughness?
- Brushes – parabolic density profile  
(E. P. K. Currie et al *Physica B*, **283** 17 – 21)
- Other scaling laws e.g. O. Guiselin *J. Phys.* **50**, 3407-3425 (1989).

We expect smooth profiles!



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# Thermoresponsive polymer brush

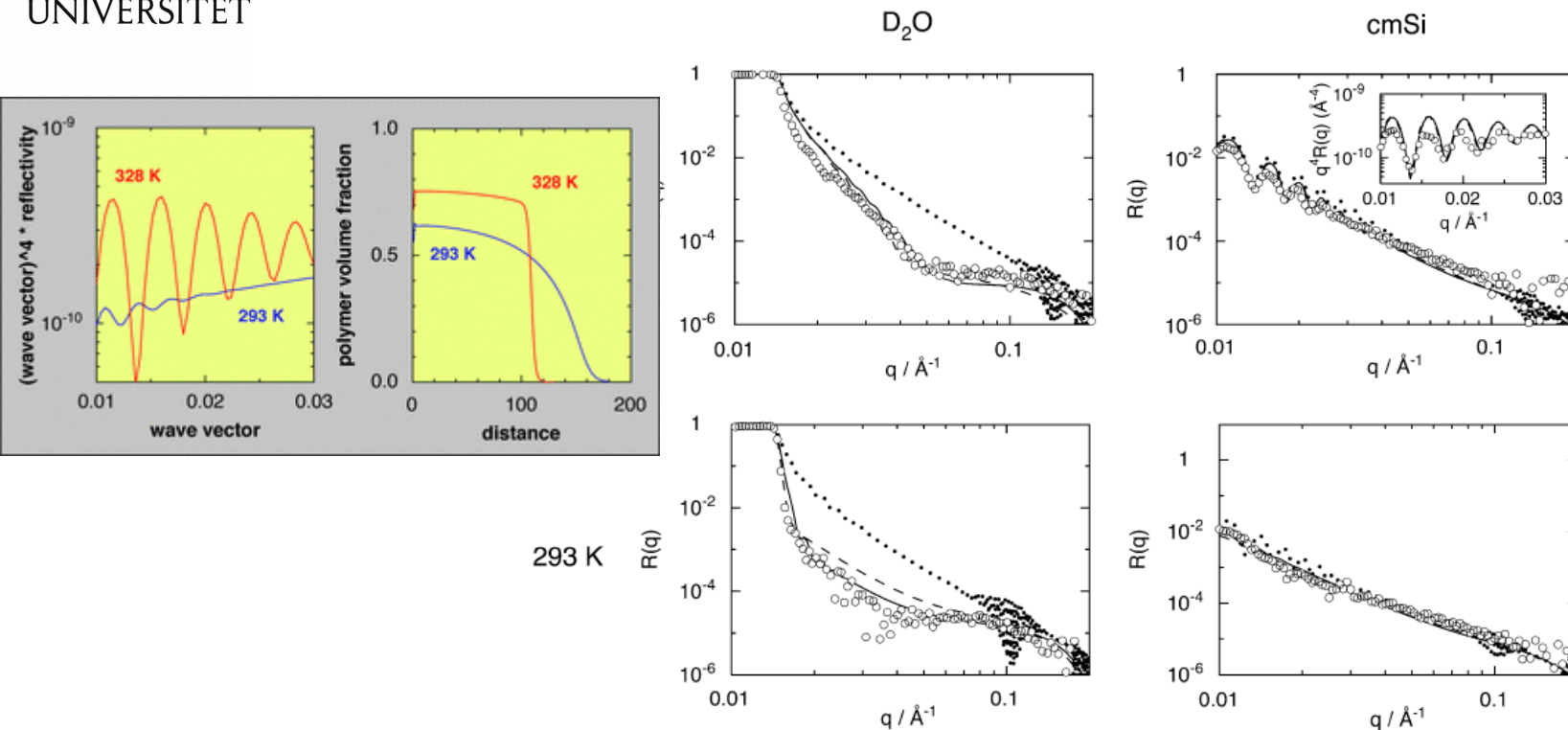


Fig. 6 Experimental reflectivity profiles obtained at ILL (circles) and fitted reflectivity profiles using a polymer layer model (dashed curves) and a lattice mean-field theory (solid curves) for polymers grafted on a Si/SiO<sub>2</sub>/initiator surface at 328 K (top) and 293 K (bottom) in D<sub>2</sub>O (left) and cmSi (right). Reflectivity profiles using a polymer layer model with zero roughness are also shown (dotted curves). The top right panel contains an inset displaying  $q^4 R(q)$  versus  $q$  for small  $q$ .

J. Zhang, et al., *Soft Matter*, 4, 500–509 (2008).



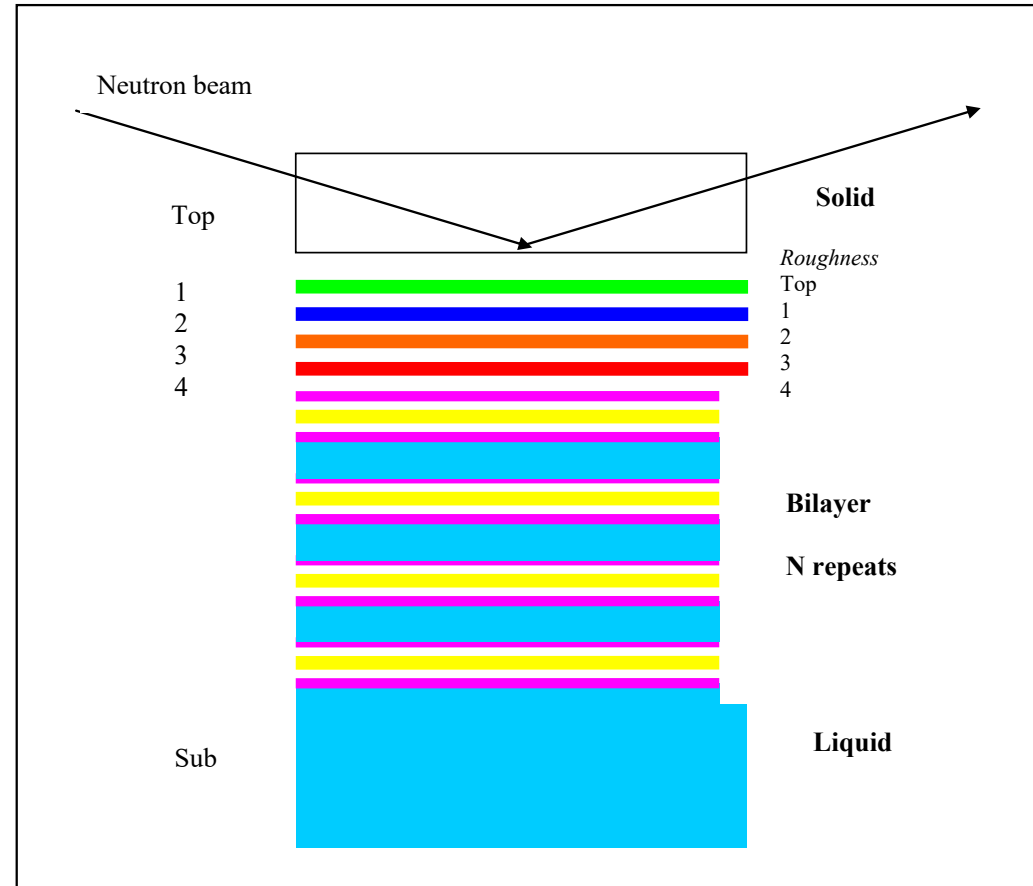
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# Repeating Layers

A one dimensional  
crystal

Bragg's law

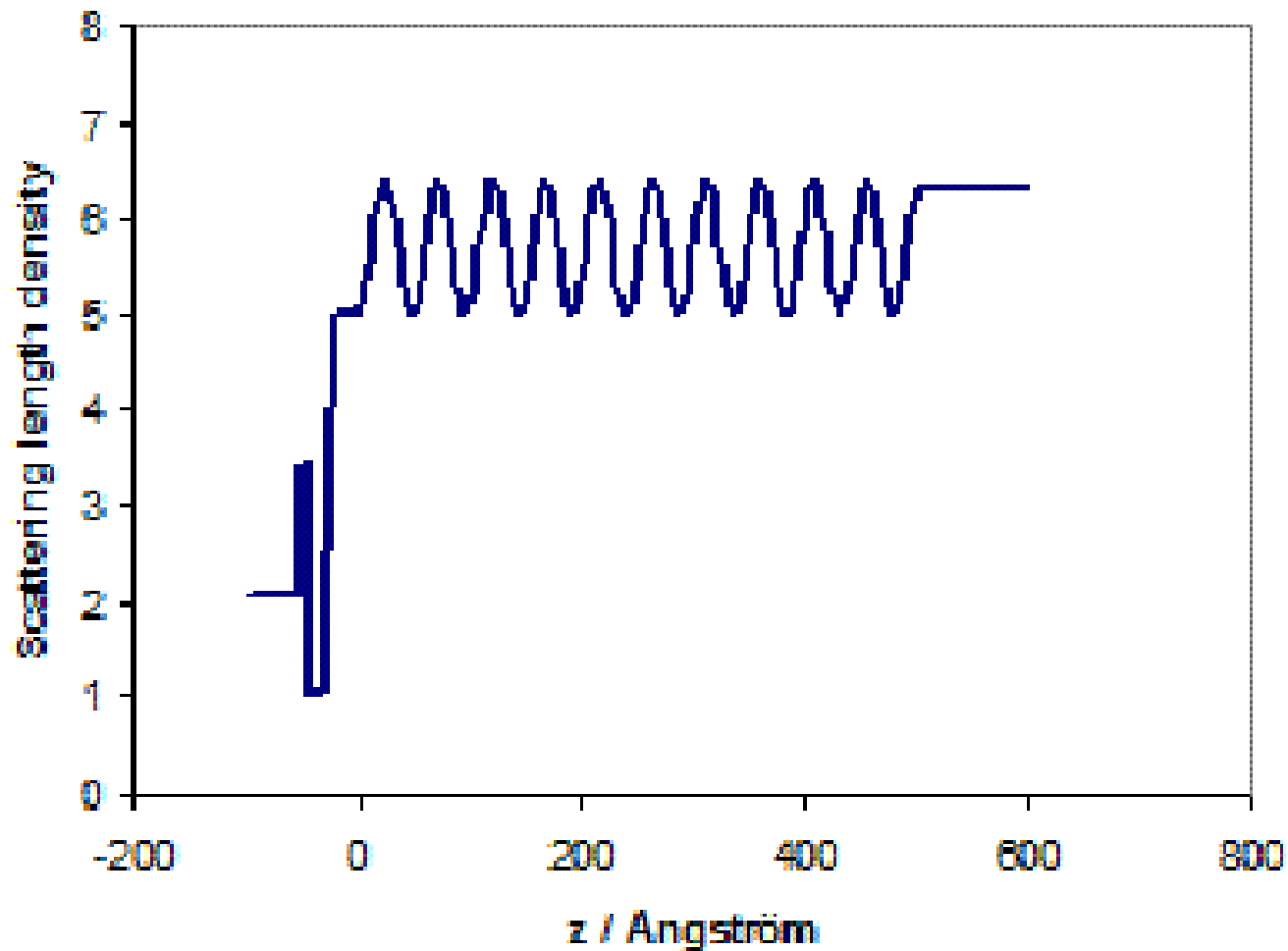
Intensity of peaks may  
Depend on size and  
disorder





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# Calculate reflectivity for a profile

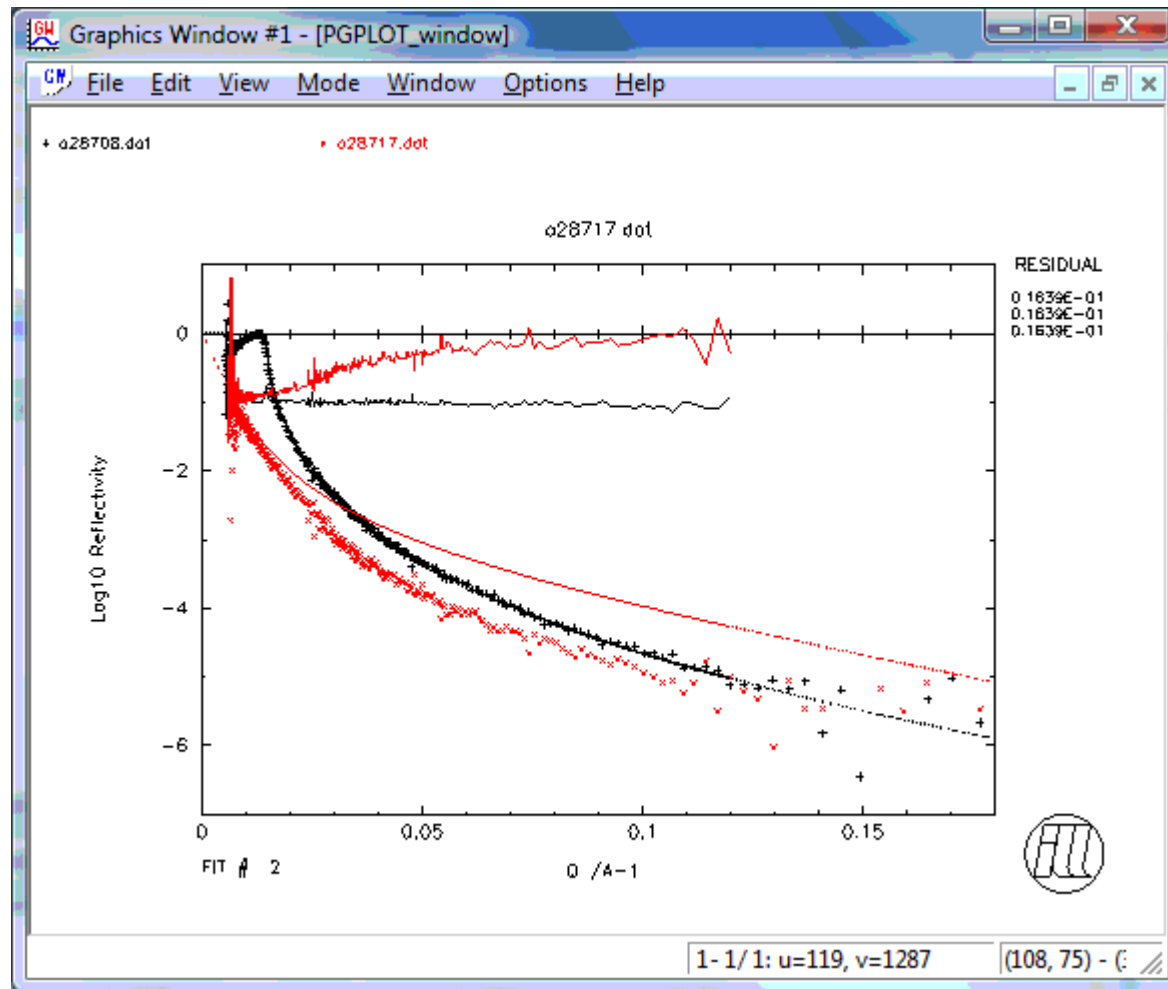






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# Using Multiple Contrasts



Simultaneous fits for multiple data sets



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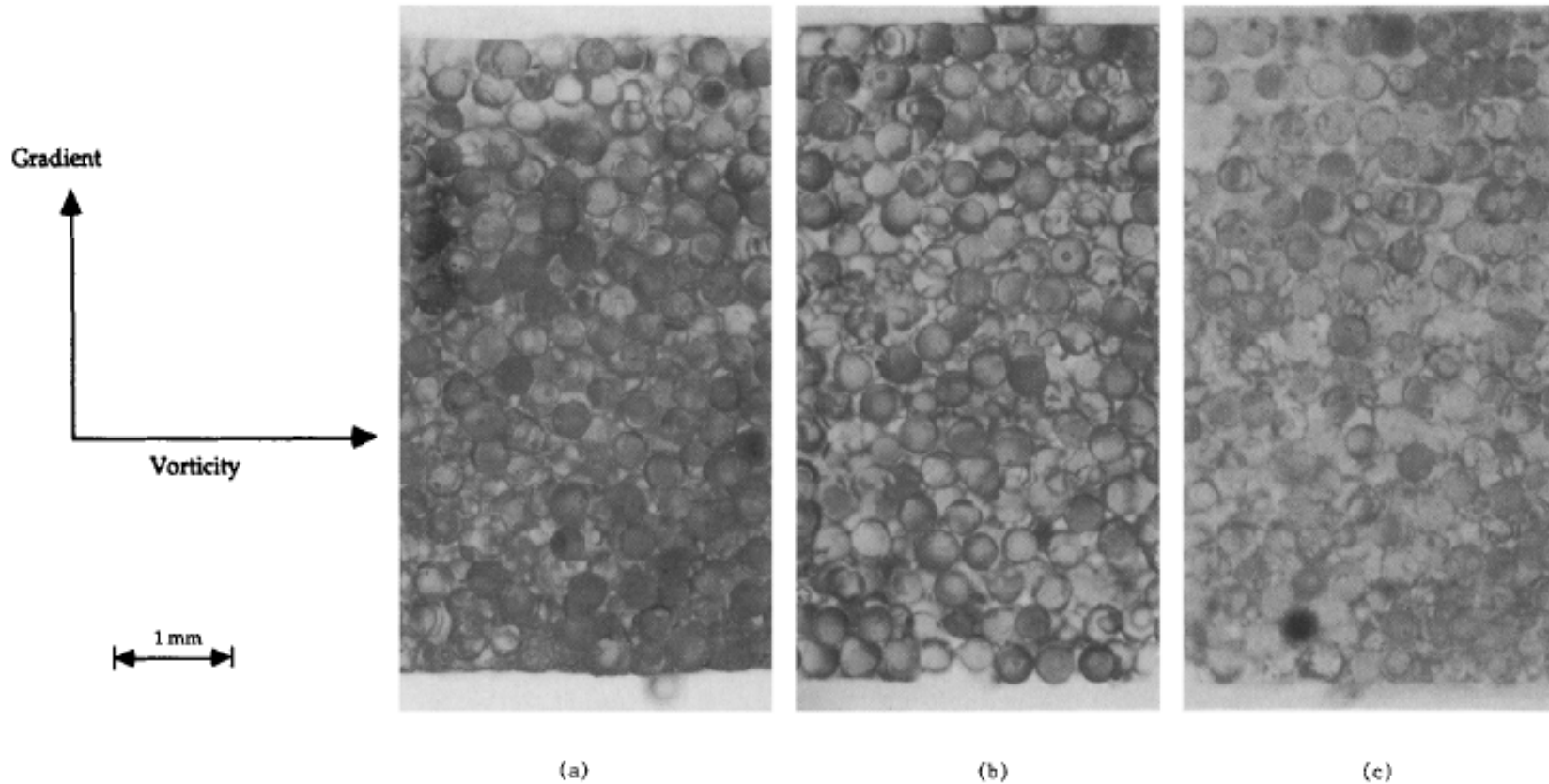
# Off-specular Scattering, GISANS, Near-surface SANS

Adrian R. Rennie



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# Interfaces are 3-dimensional



Understanding rheology – shear flow

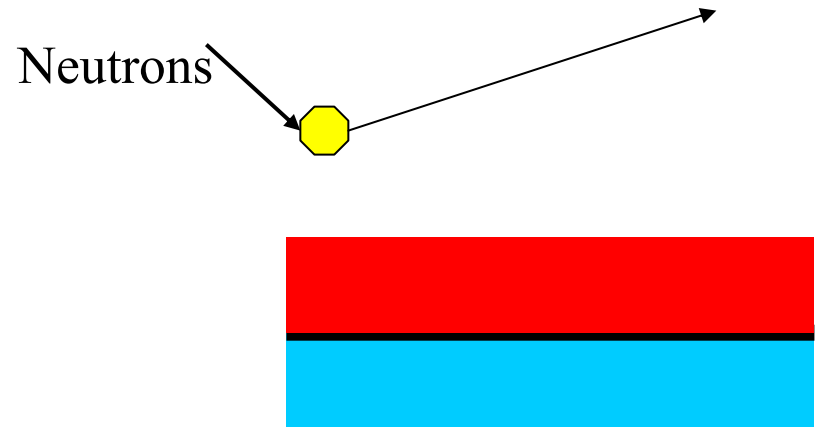
Brown et al. *Progress in Colloid and Polymer Science* **98**, (1995) 99-102.



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# Fate of a Neutron at an Interface

- Reflected
- Scattered/Diffracted from surface
- Absorbed
- Scattered from bulk (either side of surface)
- Other accidents





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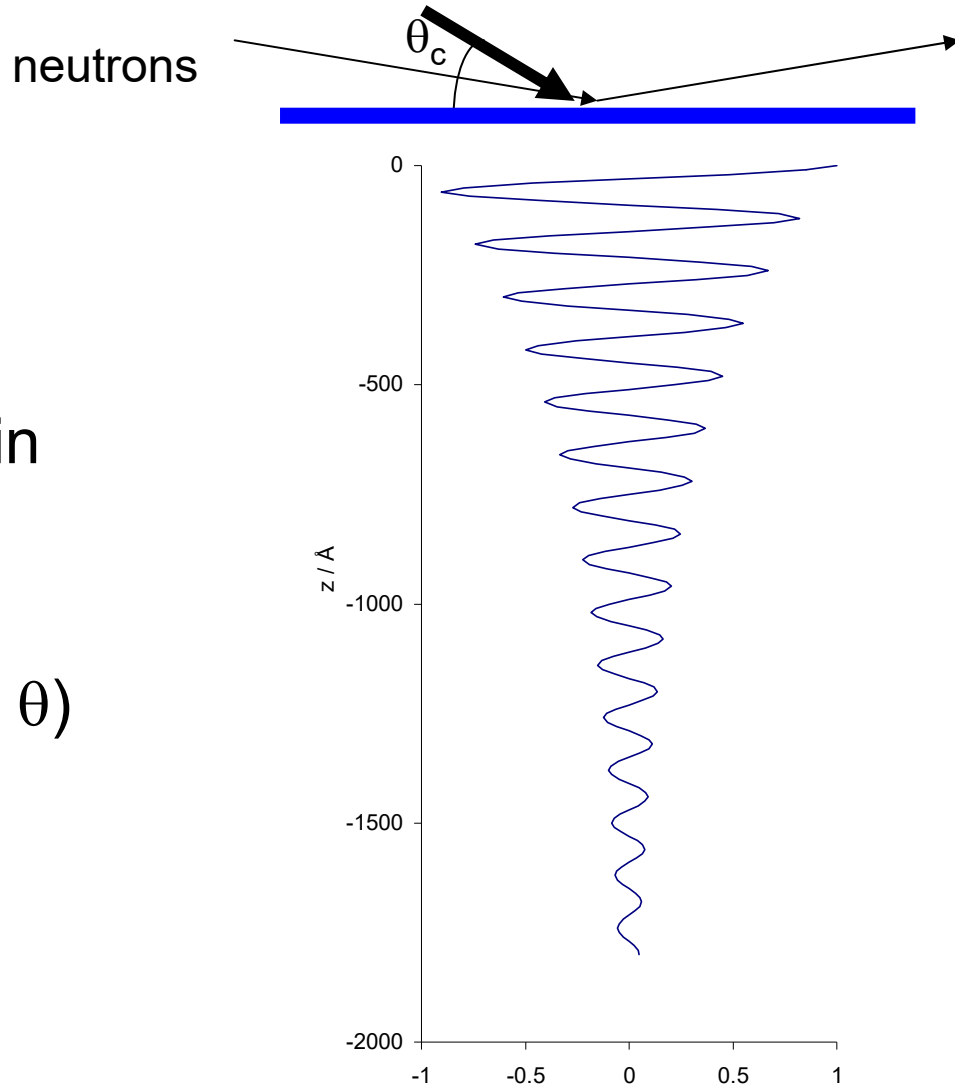
# Evanescent Wave

Below  $k_c$  no travelling wave enters the sample

Amplitude decays with depth in sample

Decay length depends on  $(\theta_c - \theta)$

Evanescent wave can cause scattering





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# Looking at Materials



Anneli Salo - Own work, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=6746303>





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# Looking at Materials

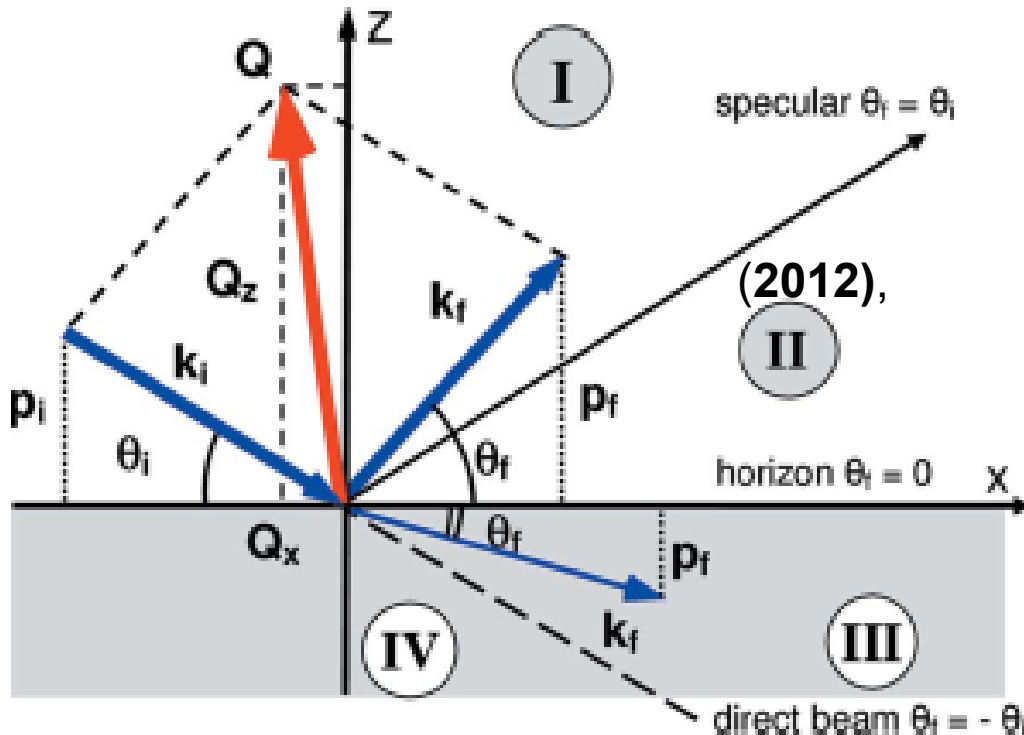


Anneli Salo - Own work, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=6746303>



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# Off-specular & Reflection



$$Q_z \approx (2\pi/\lambda) (\theta_i + \theta_f)$$

$$Q_x \approx (2\pi/\lambda) (\theta_i + \theta_f) (\theta_i - \theta_f)$$

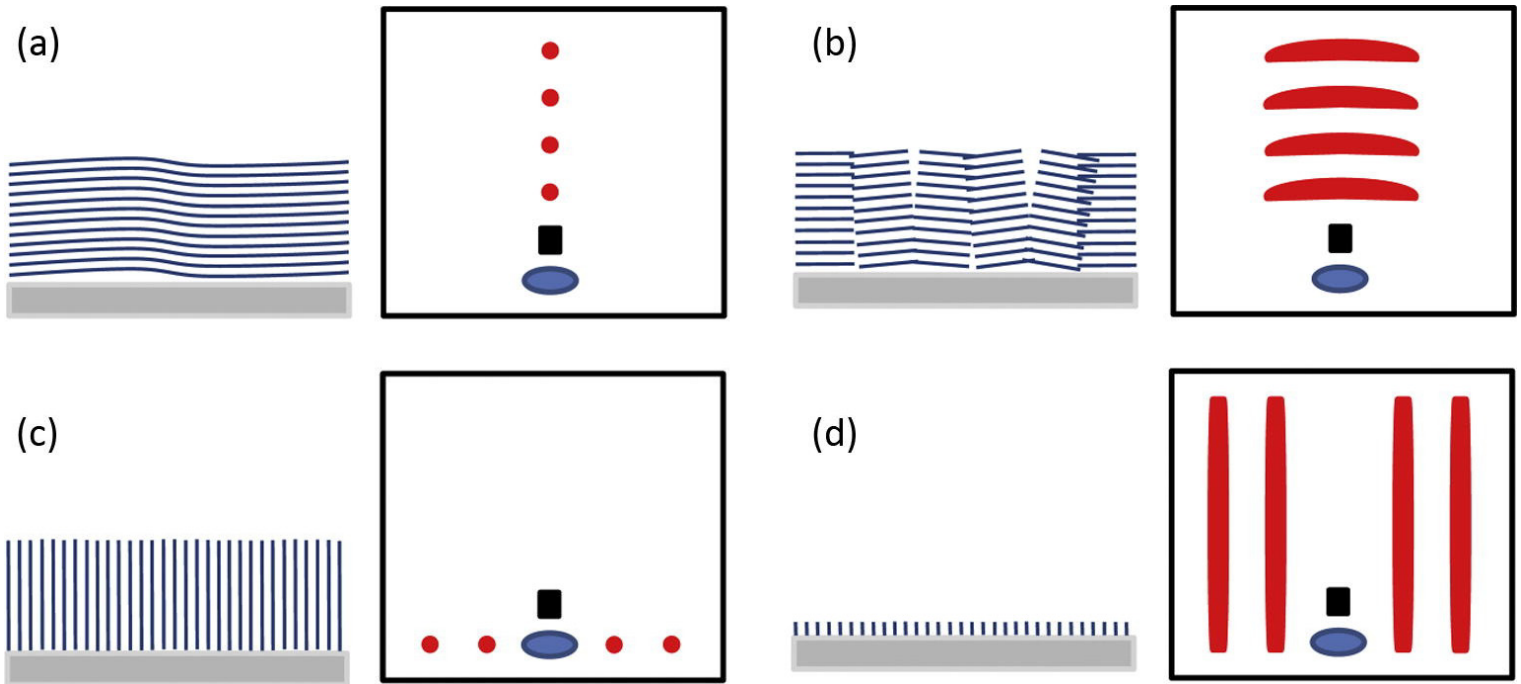
Frédéric Ott, Sergey Kozhevnikov 'Off-specular data representations in neutron reflectivity', J. Appl. Cryst. 44, (2011), 359-369.





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# Scattering from Surface Structures

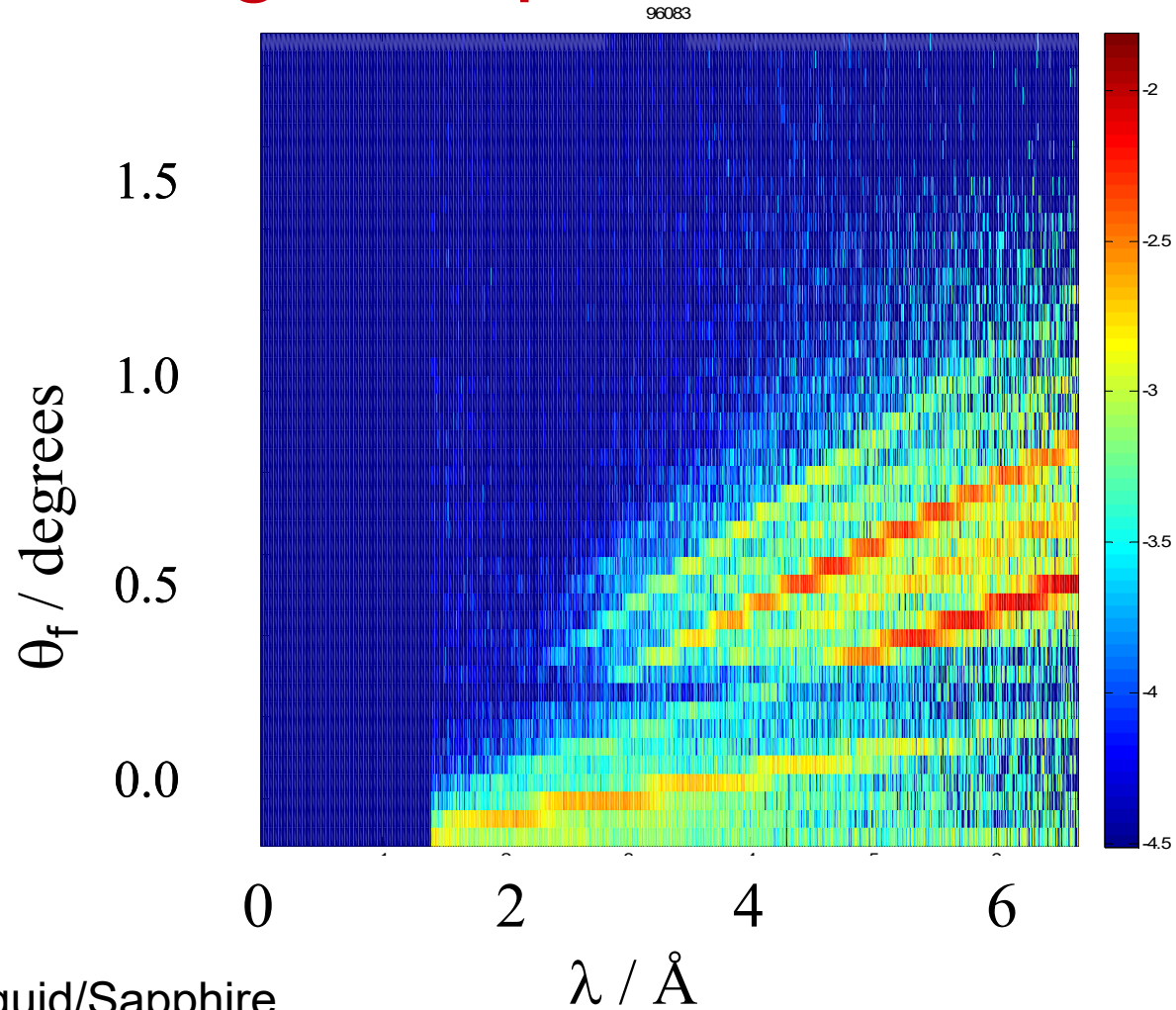


Peter Müller-Buschbaum 'GISAXS and GISANS as metrology technique for understanding the 3D morphology of block copolymer thin films' *European Polymer Journal* **81**, (2016), 470-493.



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# Strong Off-specular Scattering



PS latex in D<sub>2</sub>O Liquid/Sapphire

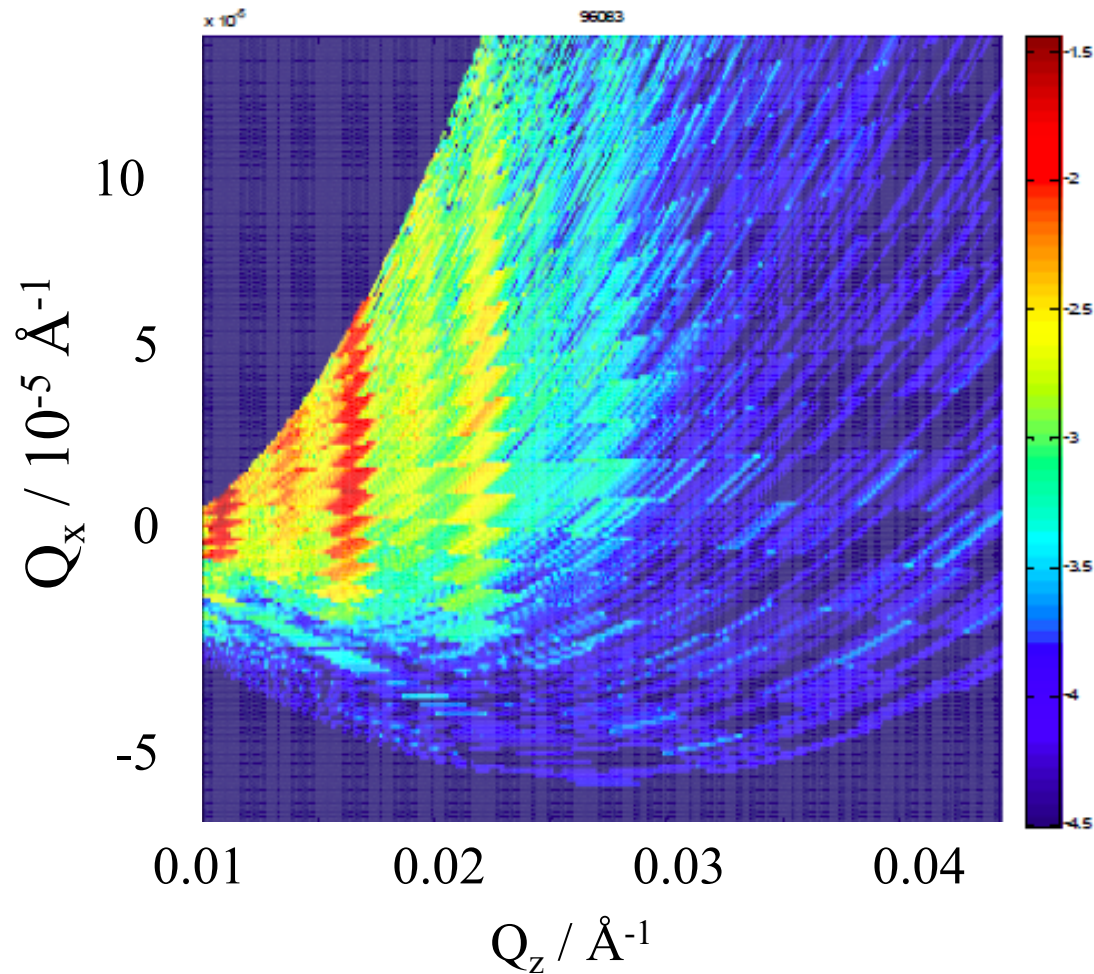
10% vol. dispersion, Radius  $\sim 350$  Å. Sapphire substrate,  $\theta_i = 0.35$  deg



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Transform to  
map of  $Q_z Q_x$

# PS latex in $D_2O$ Liquid/Sapphire



10% vol. dispersion, Radius  $\sim 350 \text{ \AA}$ , sapphire substrate,  $\theta_i = 0.35 \text{ deg}$



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# Some Scattering at Interfaces

## X-ray scattering – glass

Sinha et al., *Phys. Rev. B.* **38**, 2297, 1988.

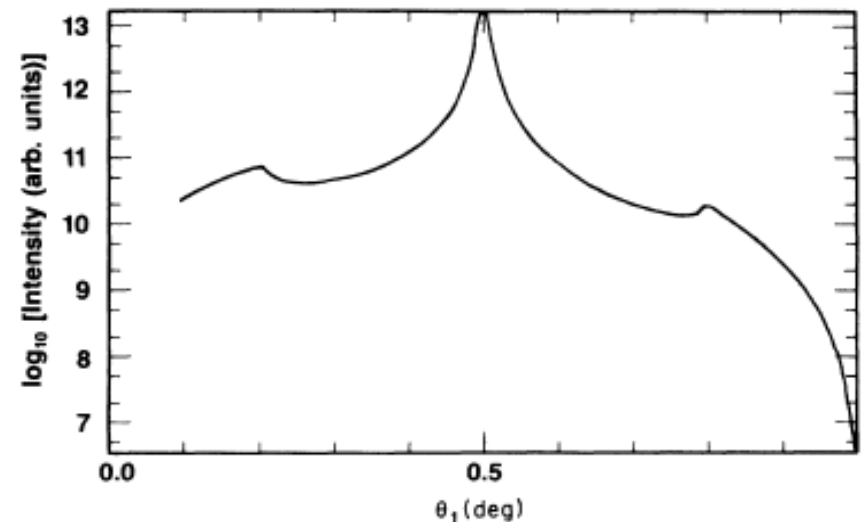
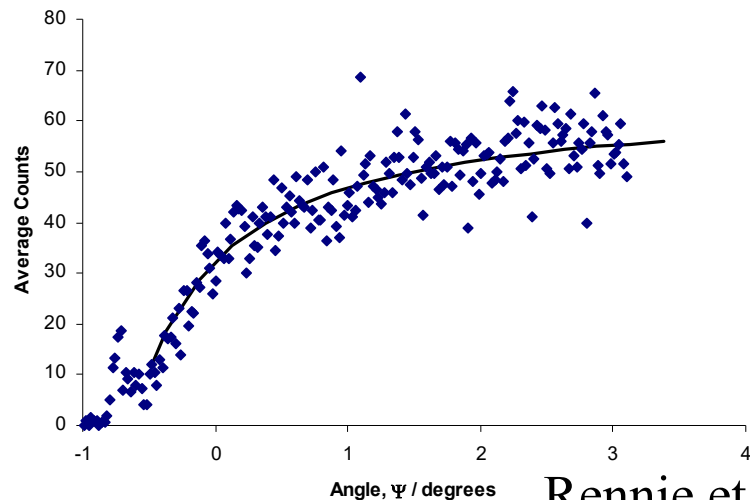
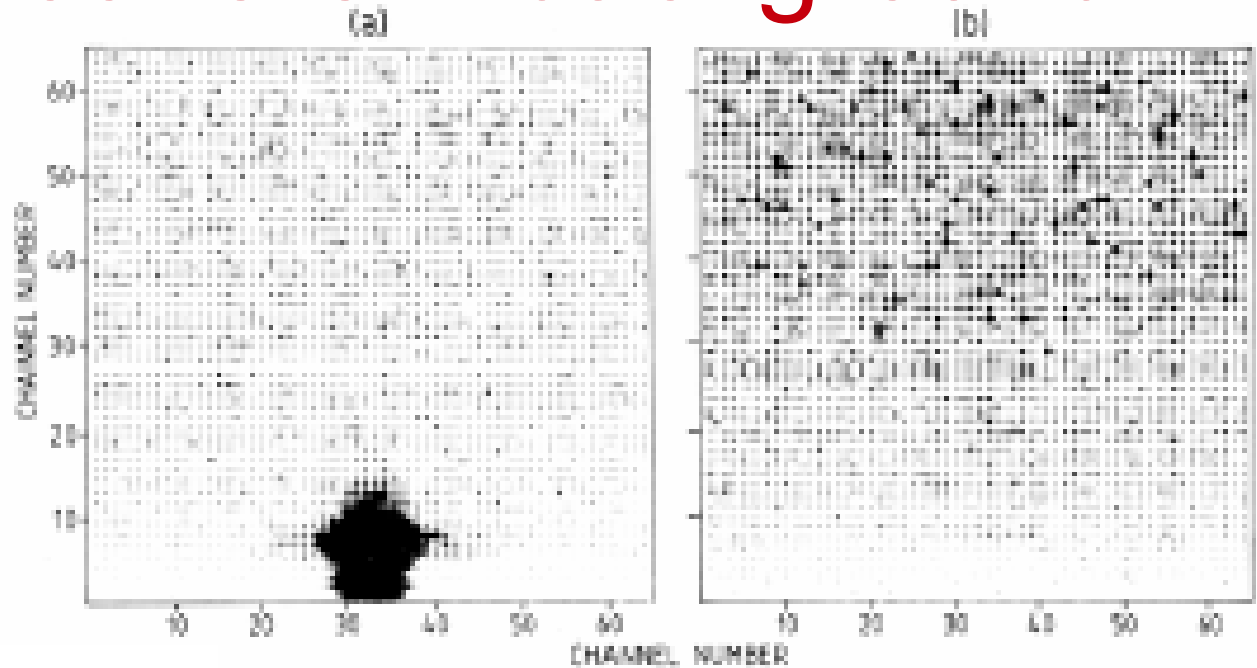


FIG. 6. Calculation of diffuse scattering in the distorted-wave Born approximation for rocking curve where  $\theta_1$  and  $\theta_2$  are varied such that  $2\theta$  is fixed at  $1^\circ$ . The asymmetry is due to the area of the illuminated surface decreasing as  $\theta_1$  is increased. The  $q_y$  direction has been integrated over. Parameters are  $\sigma = 7 \text{ \AA}$ ,  $h = 0.2$ ,  $\xi = 7000 \text{ \AA}$ , and the optical constants for Pyrex are given in Sec. V.



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# Incoherent background



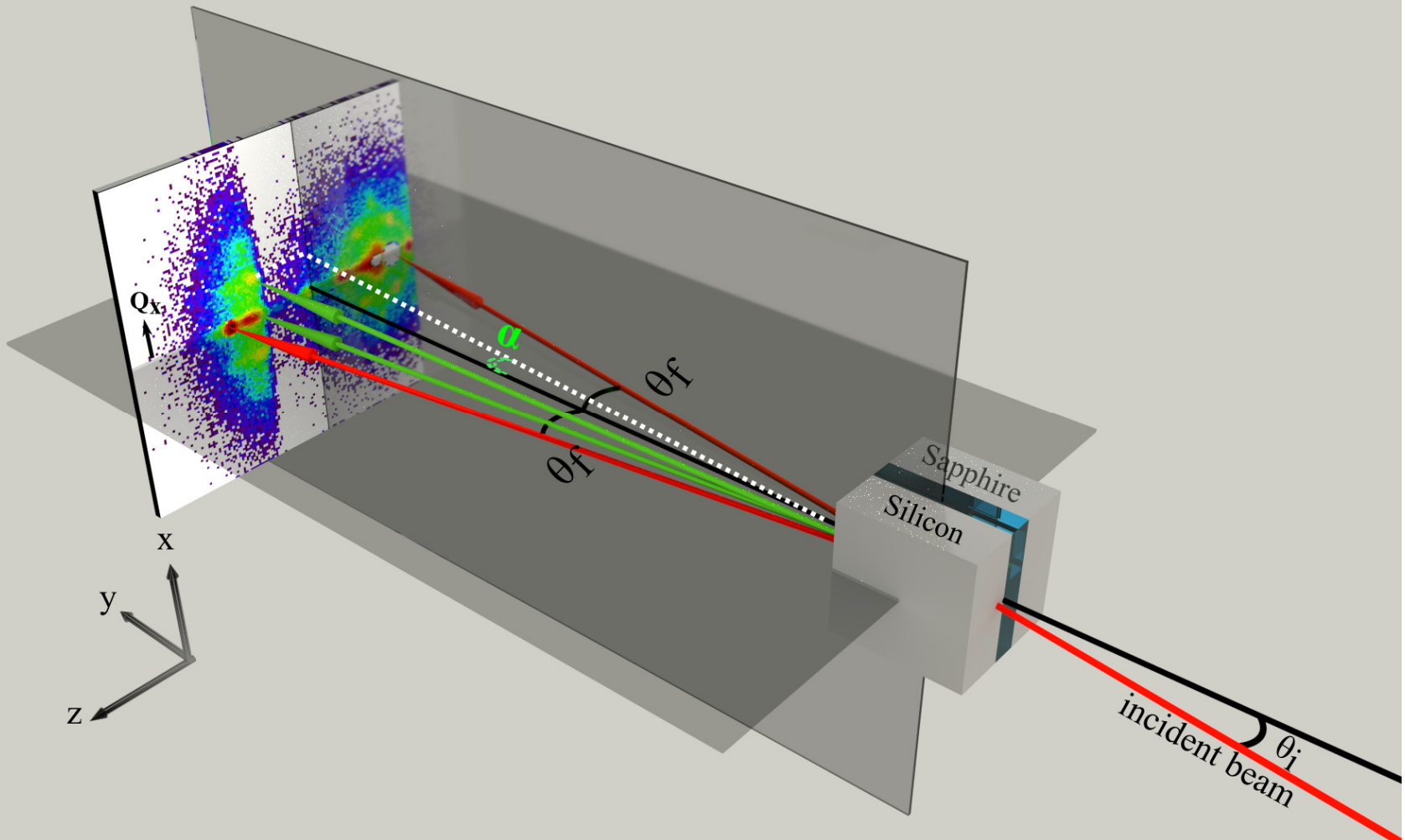
Scattering from  $D_2O$   
and from null reflecting water  
(8%  $D_2O$ )

Rennie et al., *Macromolecules* **22**, (1989), 3466-3475.



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# Interfacial structure: GISANS







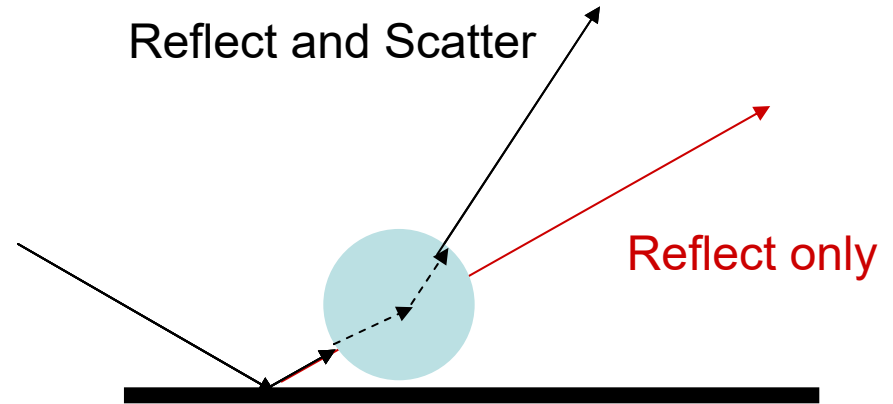
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# Calculating Scattering

## Distorted Wave Born Approximation (DWBA)

Simply allow for sequential events e.g.

Reflection then Scattering  
Refraction then Scattering  
Scattering then Reflection



Reflection followed by weak scattering.

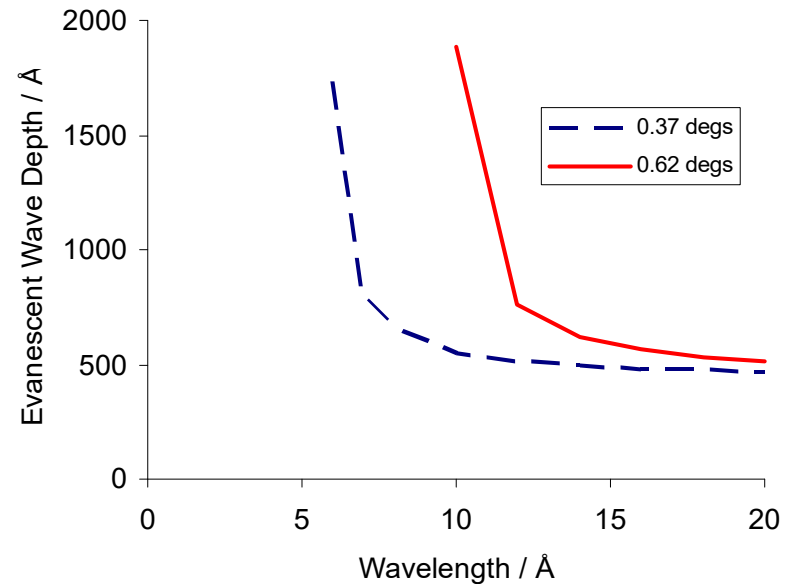
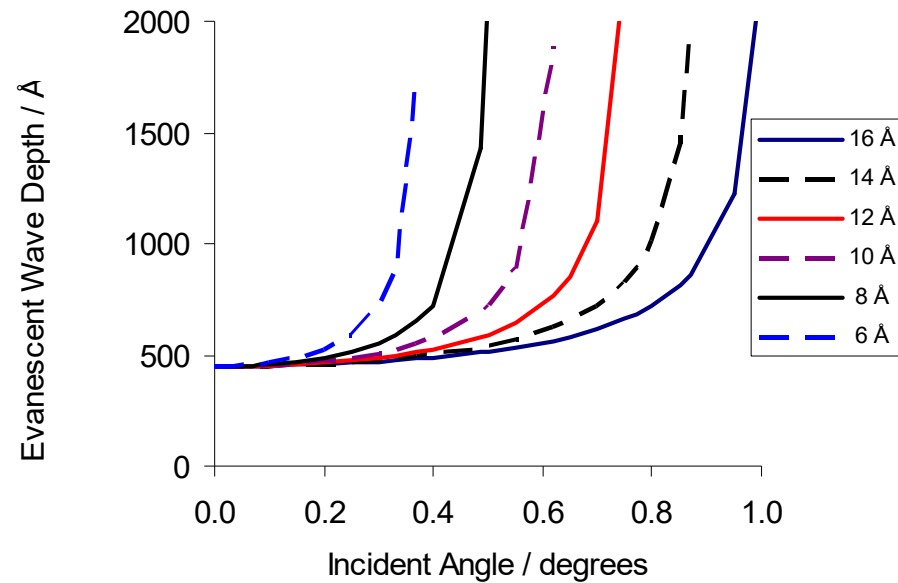
(a) Optical Matrix Calculation

(b) Weak Scattering (Born approximation)



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# How deep is the evanescent wave?



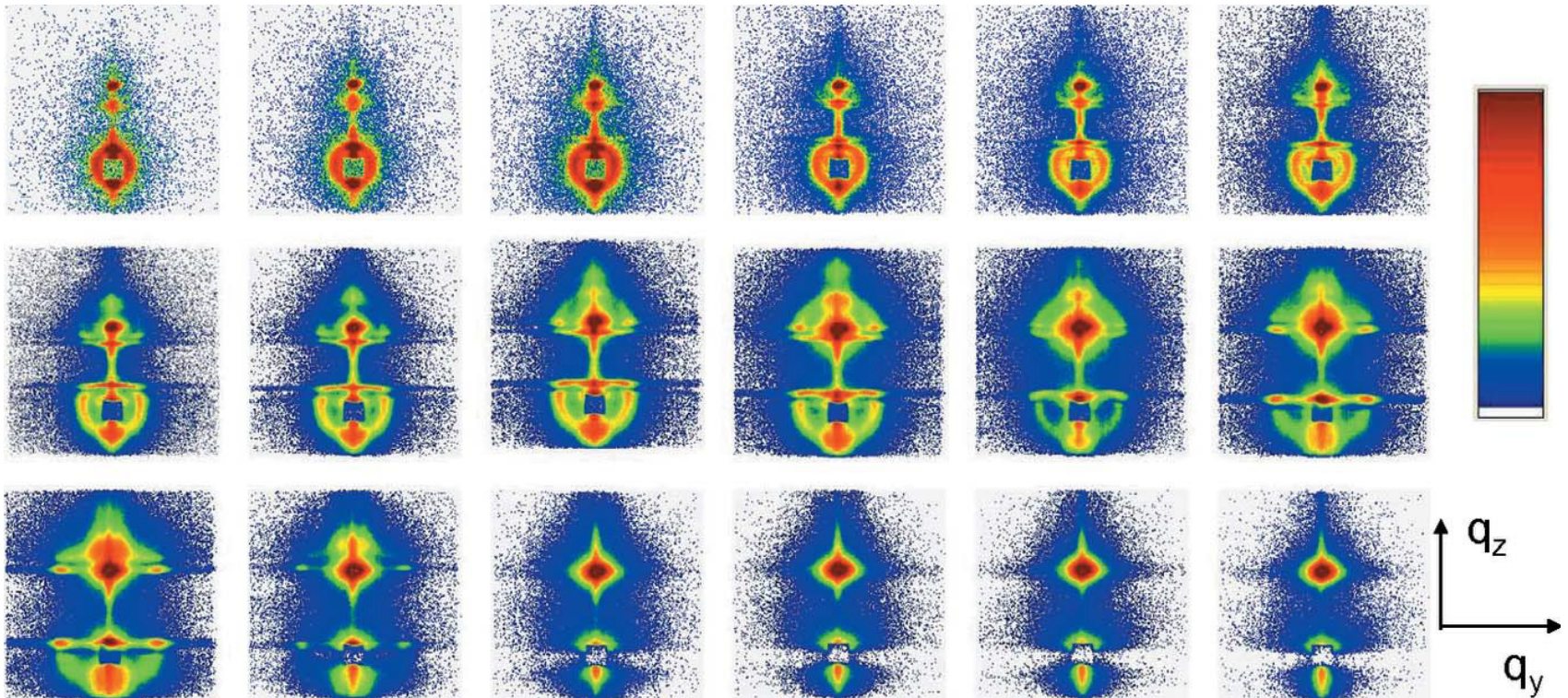
Silicon/D<sub>2</sub>O Interface





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# Copolymer films



P. Müller Buschbaum *et al.* *J. Appl. Cryst.* **47**, (2014), 1228–1237

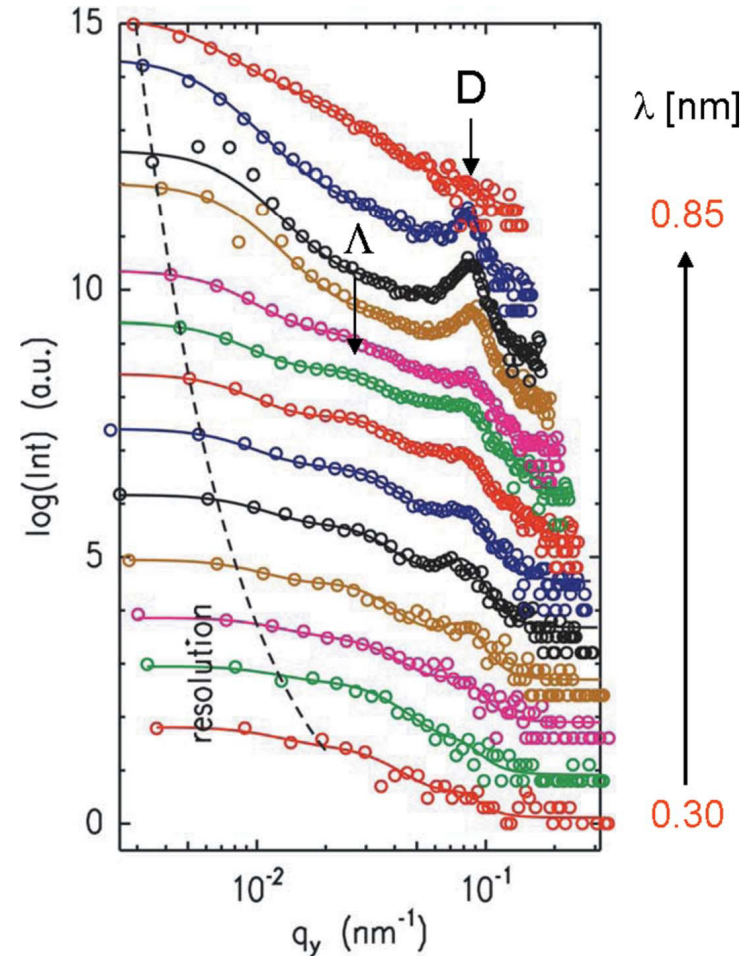


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# Changes with Depth

## Horizontal cuts

- Used wavelength to probe different depths
- Longer wavelength looks neare the surface

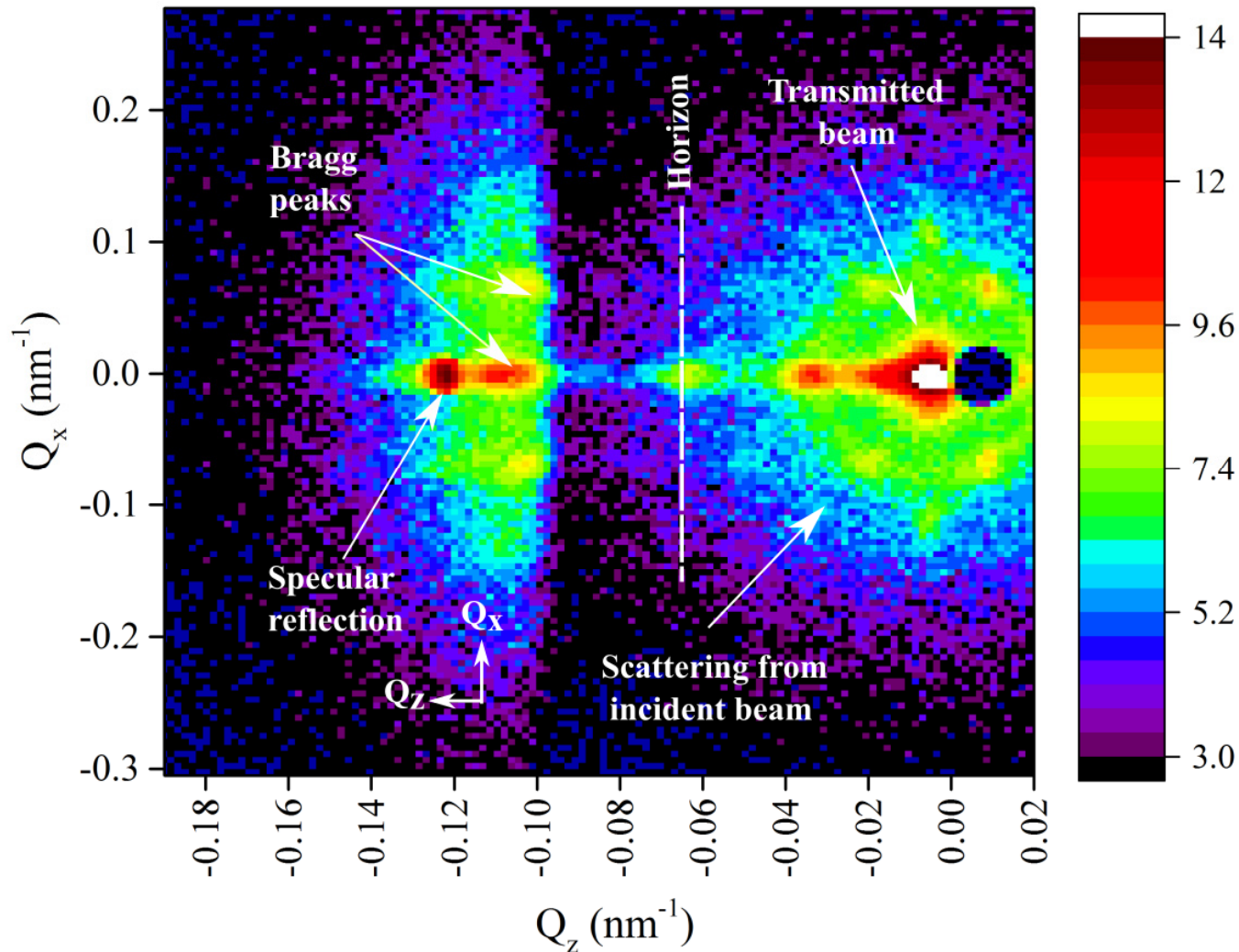


*J. Appl. Cryst.* **47**, (2014), 1228–1237



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# Diffraction from Surface Layers







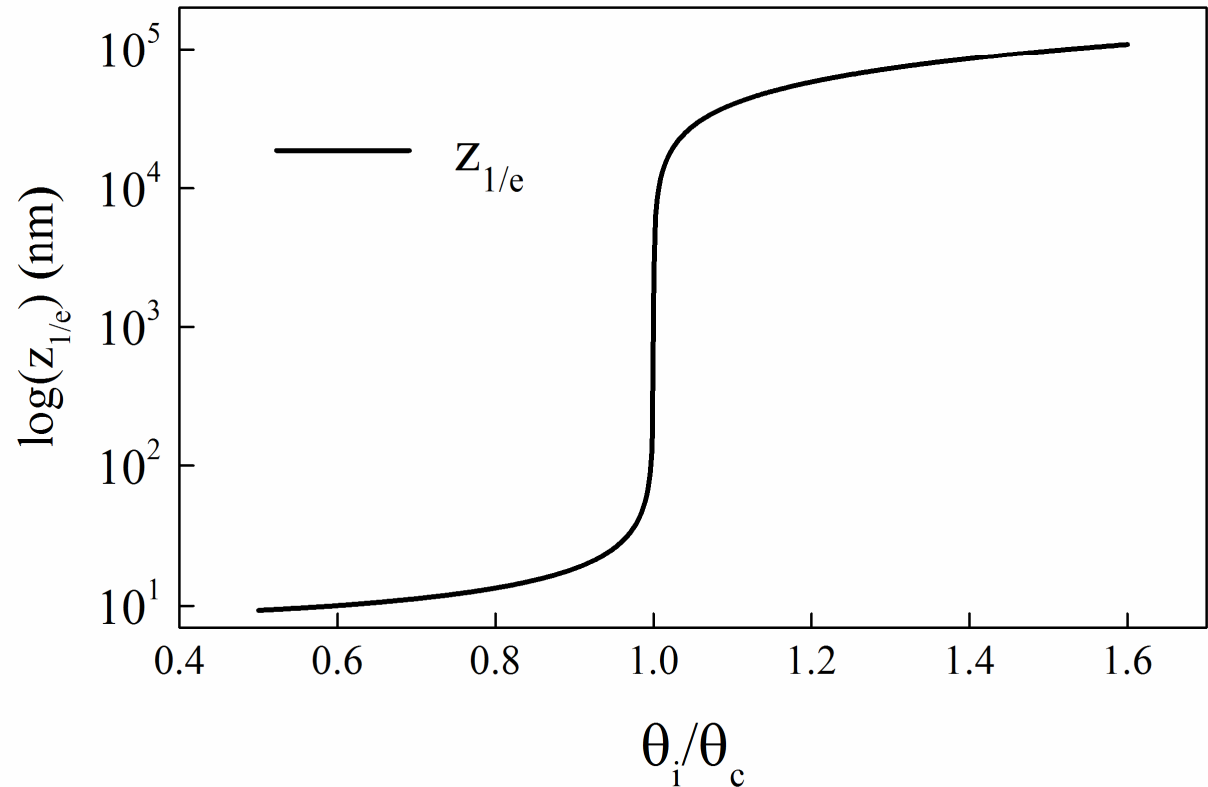
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# Penetration depth

$$z_{1/e} = \sqrt{2}\lambda / 4\pi \left[ \sqrt{(\theta_i^2 - \theta_c^2)^2 + \left(\frac{\lambda}{2\pi}\mu\right)^2} - (\theta_i^2 - \theta_c^2) \right]^{1/2}$$

A depth sensitive  
technique:

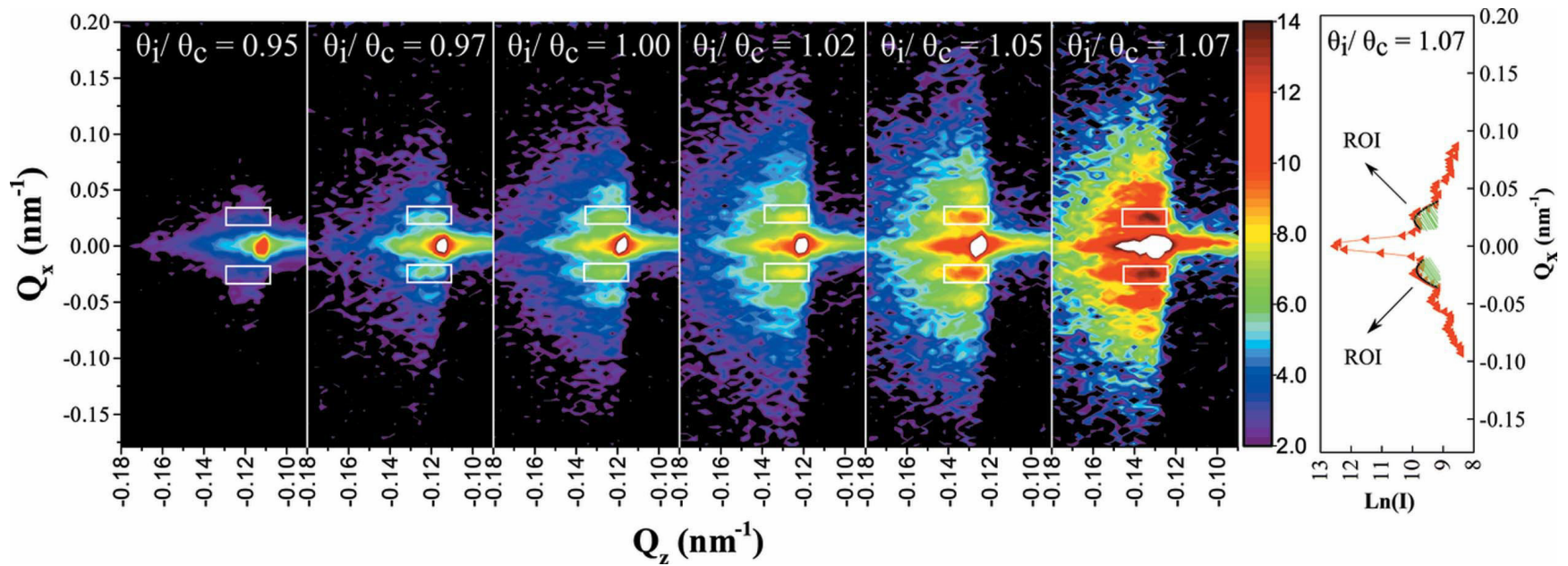
Wavelength  
Incident angle





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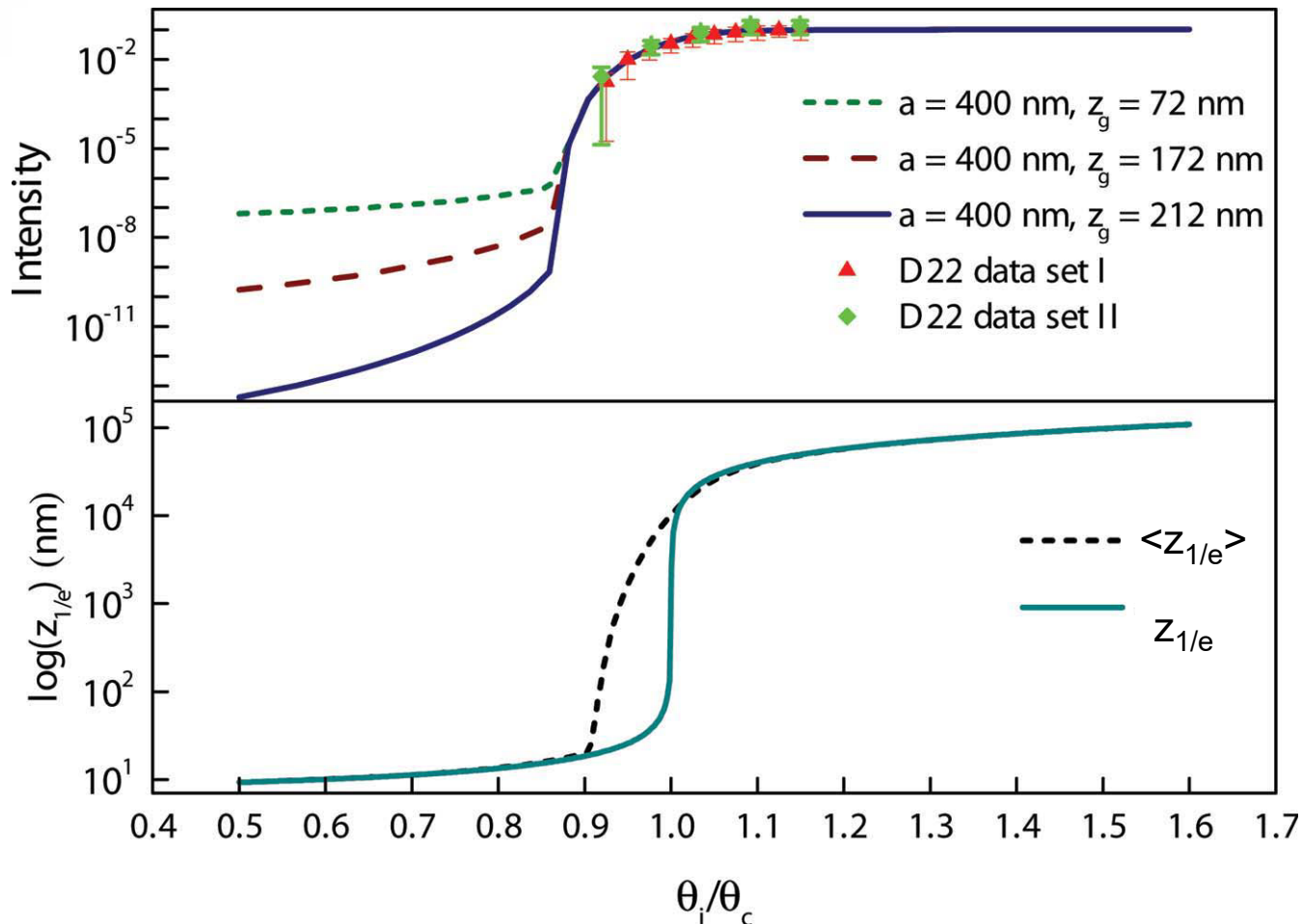
# Data at different angles

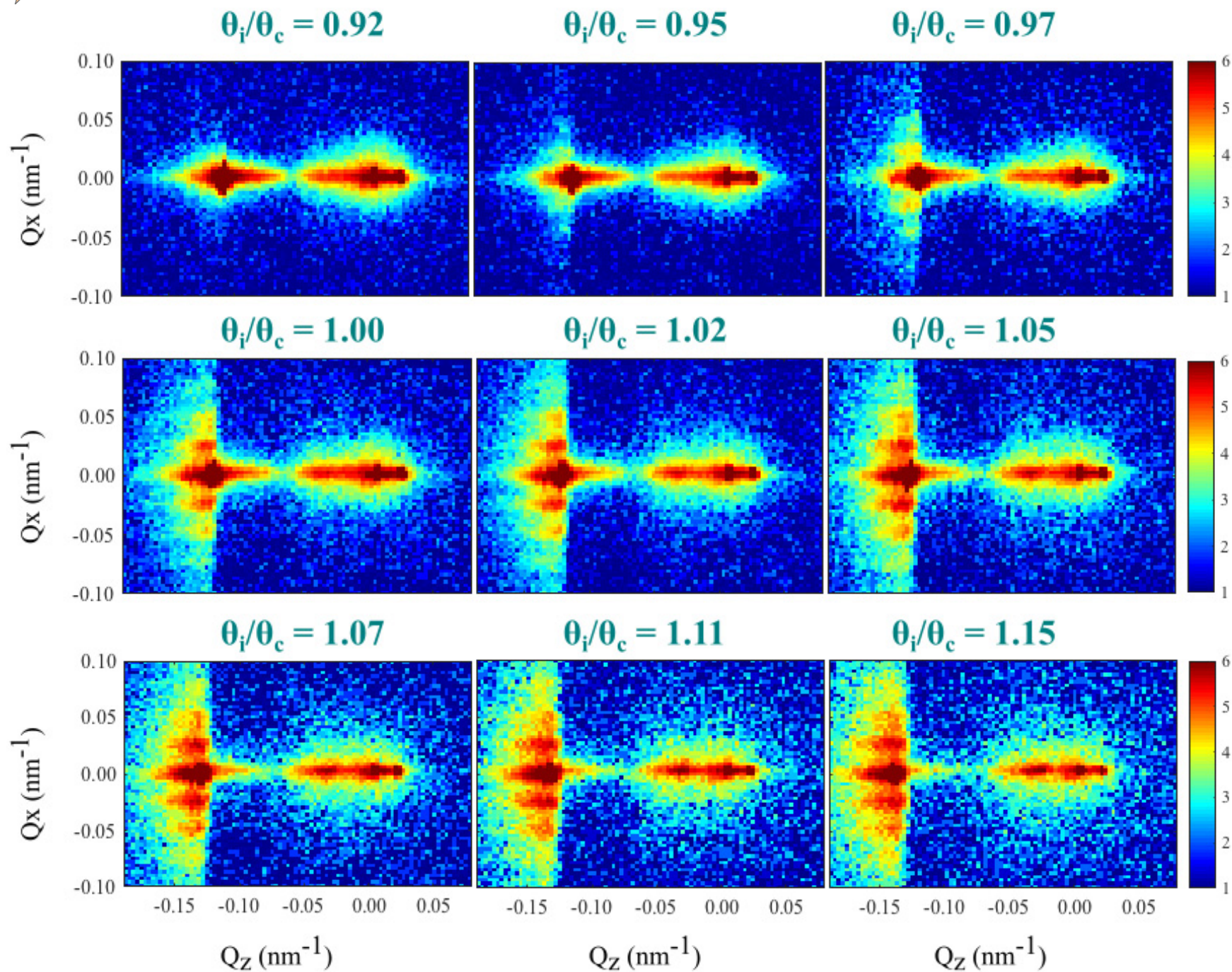




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# Data at different angles



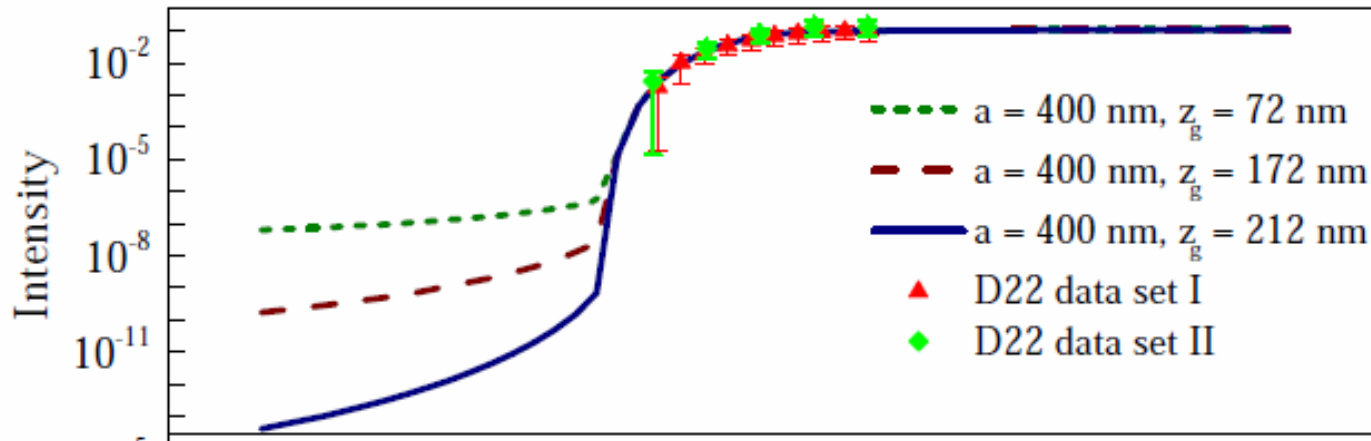






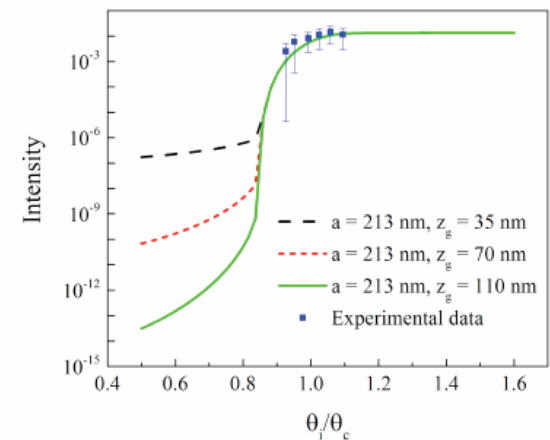
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# Calculations & Intensity Data



D22 - ILL

QCM-D data: structure forms with a separation from the interface [Hellsing et al. 2017, *manuscript*]



NG3 SANS - NCNR





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# Scattering at Interfaces

- Off-specular scattering
- Near Surface SANS
- GISANS

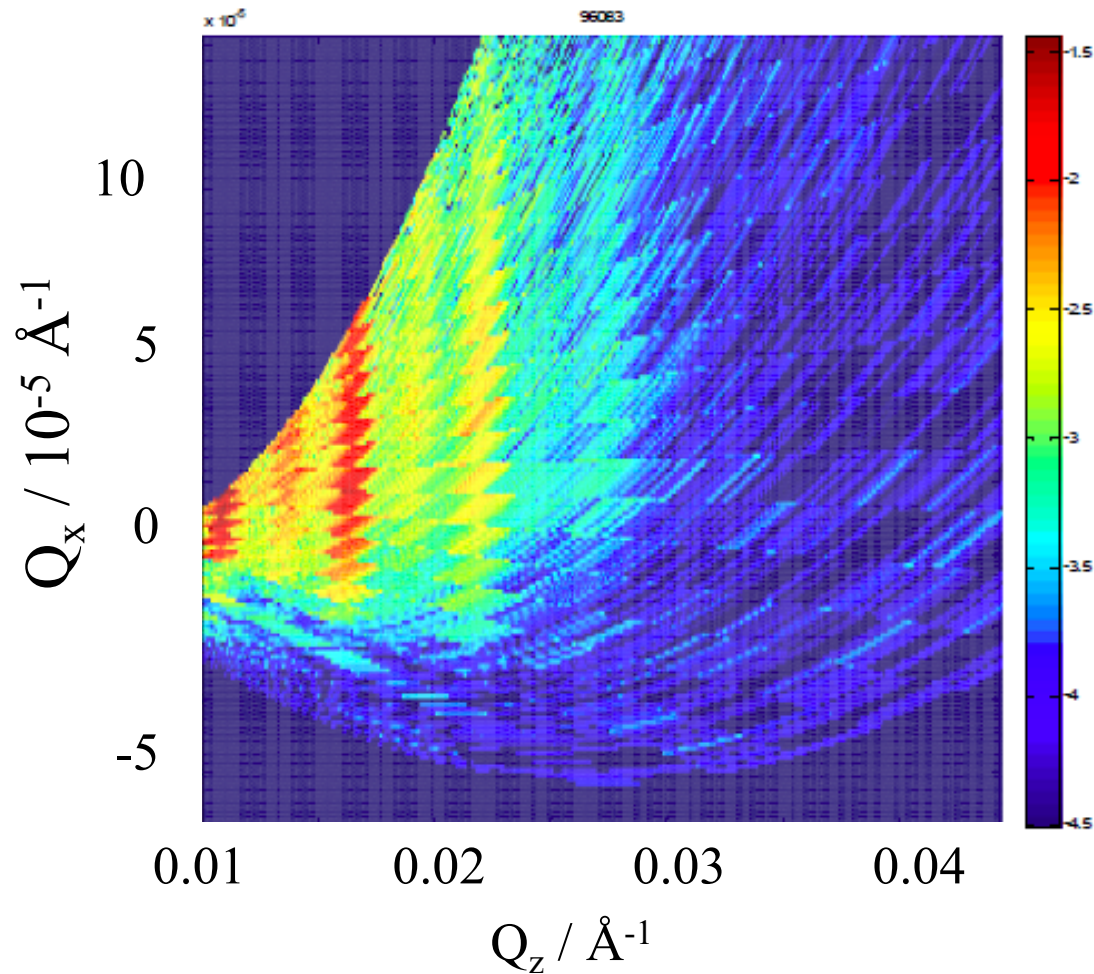
What is the difference?



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Transform to  
map of  $Q_z Q_x$

# PS latex in $D_2O$ Liquid/Sapphire



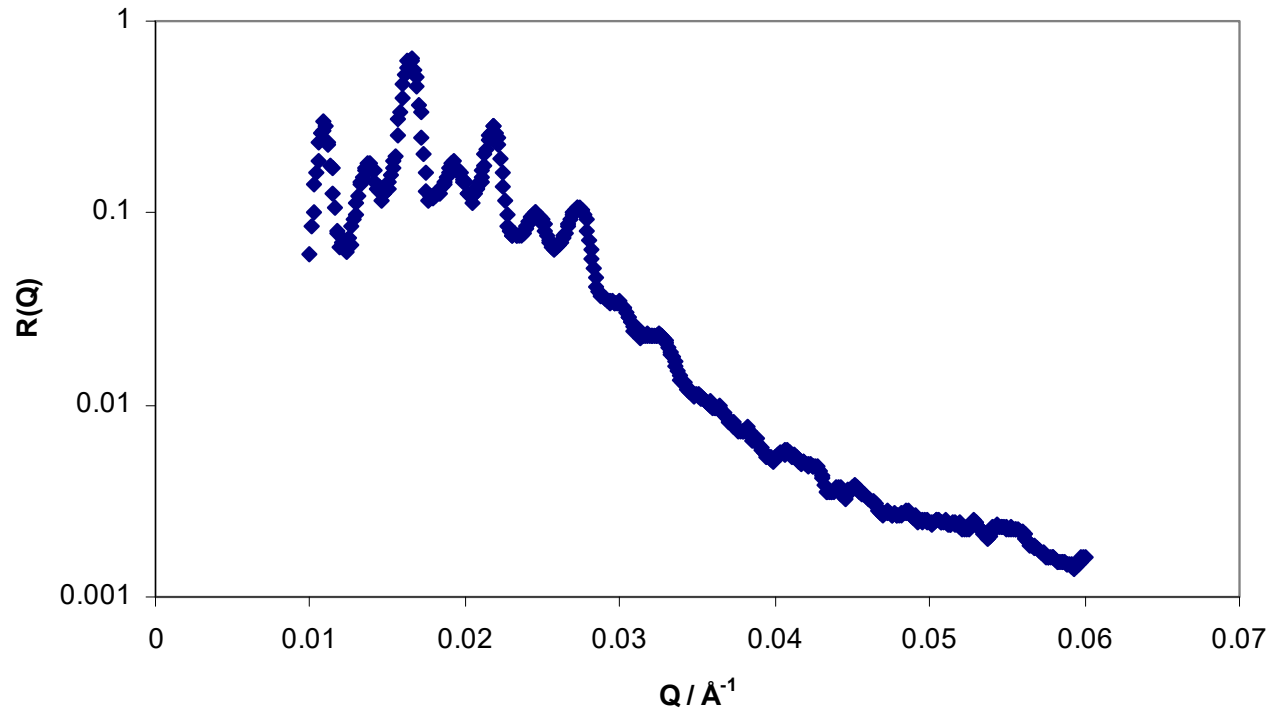
10% vol. dispersion, Radius  $\sim 350 \text{ \AA}$ , sapphire substrate,  $\theta_i = 0.35 \text{ deg}$



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# PS latex in $D_2O$ – sapphire surface

Sum along  $Q_x$



10% vol dispersion, 0.35



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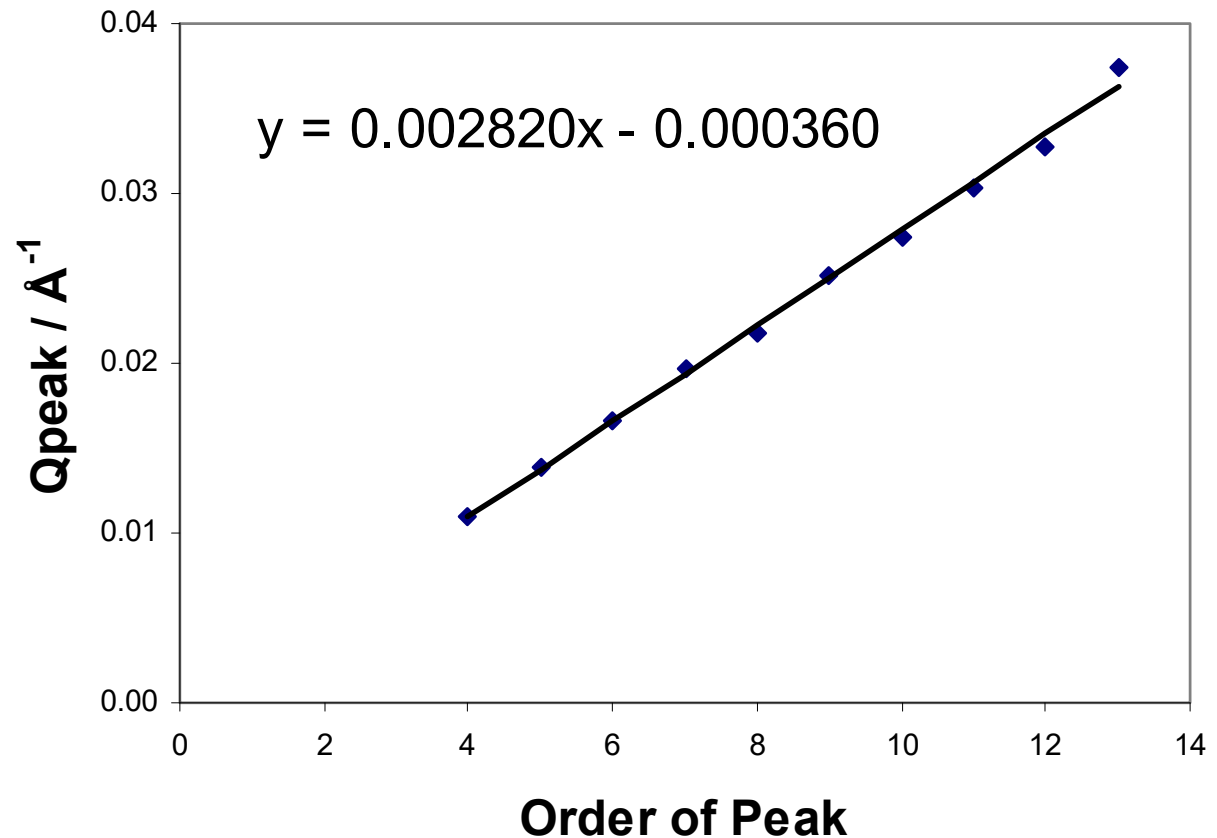
# PS latex in D<sub>2</sub>O – sapphire surface

Assign Bragg  
peaks (index)

$$Q_1 = 0.00282 \text{ \AA}^{-1}$$

$$d = 2230 \text{ \AA}$$

3 first peaks  
outside range



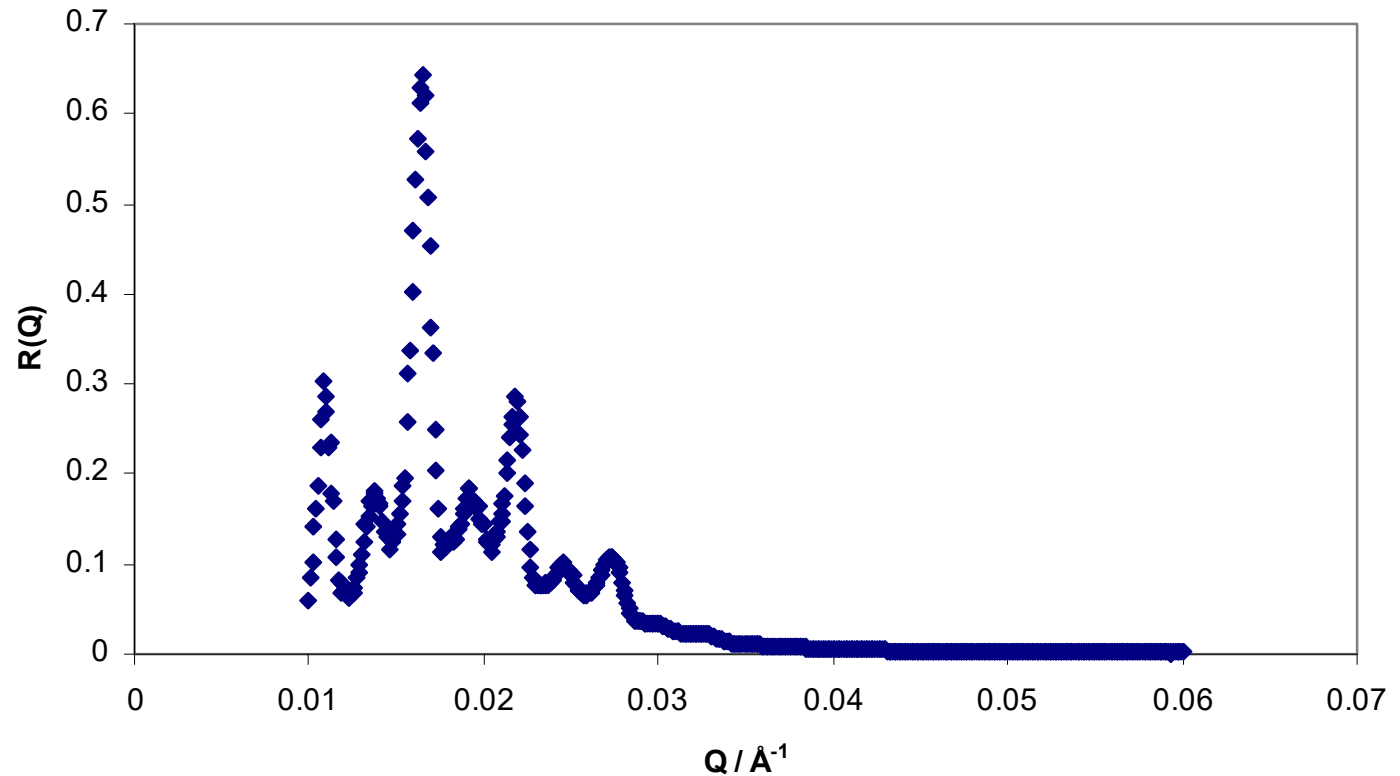
10% vol dispersion, 0.35, 0.8 and 1.5 deg



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# PS latex in D<sub>2</sub>O – sapphire surface

Sum along  $Q_x$

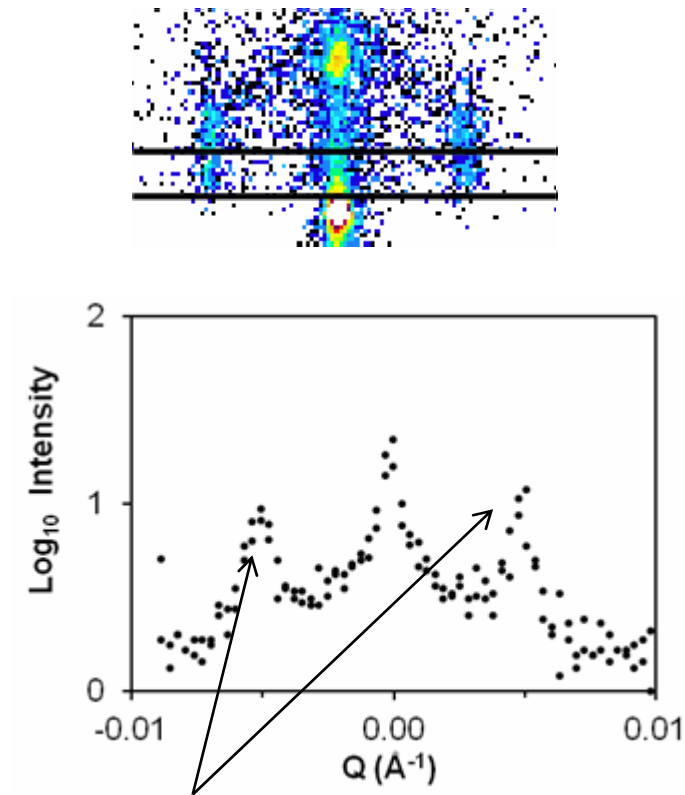


10% vol dispersion, 0.35



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# Compare $Q_x$ and $Q_z$



M. S. Hellsing, et al. *Applied Physics Letters*, **100**, (2012), 221601.