

Neutrons for Quantum Magnetism

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Conventional Magnets

Long-range magnetic order and spin-wave excitations

Origins of Quantum magnetism

low spin values, antiferromagnetic, low-dimensional, frustration, spin liquids

Neutron scattering for quantum magnets Triple Axis spectrometer, time-of-flight spectrometer

Examples of frustrated magnets

0-dimensional magnets e.g. dimer magnets

1.Dimensional magnets e.g. the spin-1/2 chain

2-Dimensional magnets e.g. Square, triangular, kagome, lattice

3-Dimensional magnets e.g. pyrochlore, spin ice and water ice

Conventional Magnetism – Magnetic Moments



- Electrons possess spin and orbital angular momenta (*s* and *I*).
- **S** and **L** for an ion can be determined by summing the electronic **s** and **I** of the unpaired electrons
- The ionic magnetic moment is $m = g_s \mu_B S$.

- S is the quantum number associated with the angular momentum S.
- S is restricted to take on discrete values either integer or half integer.



The Mn²⁺ ion, S=5/2

Conventional Magnetism - Exchange Interactions

Heisenberg interactions

$$H = \sum_{n,m} J_{n,m} \mathbf{S}_n \cdot \mathbf{S}_m$$

CuWO₄

3D magnet $|J_1| = |J_2| = |J_3| = |J_4|$ e.g. RbMnF₃



 $|J_1| = |J_2| = |J_3|, J_4 = 0$ e.g. La₂CuO₄ and CFTD

2D magnet

Anisotropic interactions

 $H = \sum -J_{n,m} \left| \varepsilon \left(\mathbf{S}_n^{\mathsf{x}} \mathbf{S}_m^{\mathsf{x}} + \mathbf{S}_n^{\mathsf{y}} \mathbf{S}_m^{\mathsf{y}} \right) + \mathbf{S}_n^{\mathsf{z}} \mathbf{S}_m^{\mathsf{z}} \right|$

Exchange interactions between magnetic ions often lead to long-range order in the ground state.

ferromagnet			antiferromagnet				spin glass										
t	t	ł	t	t	ł	ŧ	ł	o	0	0	0	0	0	0	0	0	o
ł	ł	ŧ	ŧ	ł	ł	ł	ł	0	0	0	0	-0-	0	0	0	0	0
3	- î				10		а Ч	0	0	0	0	0	0	0	0	0	Se.
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t	t	t	ł	ł	ł	ł	t	0	0	0	0	0	0	0	0	T O	0
ł	ł	t	ł	t	ţ	ł	ł	o	ø	0	0	o	0	0	0	0	0

spiral magnet



Conventional Antiferromagnets

Real Space

 Long-range magnetic order on cooling as thermal fluctuations weaken





Reciprocal Space

 Magnetic Bragg peaks appear below the transition temperatures and grow as a function of temperature



Magnetic Excitations

Real Space

 Spin-waves are the collective motion of spins, about an ordered ground state (similar to phonons)





Reciprocal Space

Observed as a well defined dispersion in energy and wavevector





The Origins of Quantum Magnetism

- Quantum fluctuations suppress long-range magnetic order, spinwave theory fails
- Quantum effects are most visible in magnets with
 - low spin values
 - antiferromagnetic exchange interactions
 - low-dimensional interactions
 - frustrated interactions
- Quantum effects give rise to exotic states and excitations

$$H = \sum_{n,m \neq n} H_{n,m}$$
$$H_{n,m} = J_{n,m} S_n S_m = J_{n,m} \left(S_n^x S_m^x + S_n^y S_m^y + S_n^z S_m^z \right)$$
$$H_{n,m} = J_{n,m} S_n^z S_m^z + J \left(S_n^+ S_m^- + S_n^- S_m^+ \right)$$



$$H_{n,m} = J_{n,m}S_{n}^{z}S_{m}^{z} + J(S_{n}^{+}S_{m}^{-} + S_{n}^{-}S_{m}^{+})$$

- Fluctuations have the largest effect for low spin values
- For S=1/2, changing S^z by 1 unit reverses the spin direction





1-1-1-1-1-1-1-1-1-1-1-1-1-1-1

• Parallel spin alignment is an eigenstate of the Hamiltonian and the ground state of a ferromagnet.

• Antiparallel spin alignment (Néel state) is not an eigenstate of the Hamiltonain and is not the true ground state of an antiferromagnet.

J>0 ferromagnetic $H_{1,2} = J(S_1^+S_2^- + S_1^-S_2^+ + S_1^zS_2^z)$ and

J>0 antiferromagnetic

 $\begin{array}{c} \int J \\ S_{1,2} |\uparrow_{1}\uparrow_{2}\rangle = J/4 |\uparrow_{1}\uparrow_{2}\rangle \\ H_{1,2} |\uparrow_{1}\downarrow_{2}\rangle = -J/4 |\uparrow_{1}\downarrow_{2}\rangle + J/4 |\downarrow_{1}\uparrow_{2}\rangle \\ H_{1,2} |\uparrow_{1}\downarrow_{2}\rangle = -J/4 |\uparrow_{1}\downarrow_{2}\rangle + J/4 |\downarrow_{1}\uparrow_{2}\rangle \\ S_{2}=1/2 \\ S_{2}=-1/2 \\ S$

Low-Dimensional Interactions

For three-dimensional magnets each magnetic ion has six neighbours For a one-dimensional magnet there are only two neighbours Neighbouring ions stabilize long-range order and reduce fluctuations

$$H_{n,m} = J_{n,m}S_n^{z}S_m^{z} + J(S_n^{+}S_m^{-} + S_n^{-}S_m^{+})$$



1D S=1/2



Frustrated Interactions

- In some lattices with antiferromagnetic interactions it is impossible for the spins to satisfy all the bonds simultaneously, this phenomenon is known as a geometrical frustration.
- Long-range order is suppressed as the spins fluctuate between the different degenerate configurations. ?



Anisotropy produces frustration if the anisotropy is incompatible with the spin direction favoured by the interactions



Examples of Quantum Magnets

- Quantum magnets are characterised by suppression of magnetic order, $T_N << T_{CW}$ and <S> < S, in some cases the magnet never orders.
- The excitations are broadened and renormalised with respect to spinwave theory, and can be characterised by different quantum numbers.
- New theoretical approaches are required to understand these systems

Spin liquids,

 no local order, no static magnetism highly entangled, dynamic ground state, topological order, spinon excitations



Magnetic order is suppressed therefore most information about quantum magnets comes from their excitations.

It is important to resolve the excitations as a function of energy

- inelastic neutron scattering.

Since the excitations are often diffuse

- wide detector coverage is useful



$$\frac{d^2\sigma}{d\Omega dE} = \frac{k_f}{k_i} \left(\frac{m}{2\pi\hbar^2}\right)^2 \sum_{\rho_i, s_i} p_{\lambda_i} p_{s_i} \sum_{\rho_f, s_f} \left| \left\langle \boldsymbol{k}_f \boldsymbol{s}_f \boldsymbol{\rho}_f \left| \boldsymbol{V} \right| \boldsymbol{k}_i \boldsymbol{s}_i \boldsymbol{\rho}_i \right\rangle \right|^2 \delta\left(\boldsymbol{E}_{\rho_i} - \boldsymbol{E}_{\rho_f} + \hbar\omega \right) \quad \boldsymbol{E} = \hbar\omega = \frac{\hbar^2}{2m} \left(k_i^2 - k_f^2 \right)$$

V - the magnetic interaction between neutron and electrons

The electrons in an atom possess spin and orbital angular momentum, both of which give rise to an effective magnetic field. The neutrons interact with this field because they possess a spin moment

The interaction between a neutron at point **R** away from an electron with momentum **I** and spin **s** is

$$V_{magnetic} = -\mu_n \cdot B = \frac{-\mu_0 \gamma \mu_N 2 \mu_B}{4\pi} \sum_j \sigma \cdot \left\{ curl\left(\frac{s_j \times \hat{R}_j}{R^2}\right) + \frac{1}{\hbar} \left(\frac{l_j \times \hat{R}_j}{R^2}\right) \right\}$$

$$V_{nuclear} = \frac{2\pi\hbar}{m} \sum_j b_j \delta\left(r - r_j\right)$$
Origin

The Magnetic Cross-section

Cross section for spin only scattering by ions

$$\left(\frac{d^{2}\sigma}{d\Omega dE'}\right) = \frac{\left(\gamma r_{0}\right)^{2}}{2\pi\hbar} \frac{k_{f}}{k_{i}} \left[F\left(\boldsymbol{Q}\right)\right]^{2} \exp\left\langle-2W\right\rangle \sum_{\alpha,\beta} \left(\delta_{\alpha,\beta} - \hat{Q}_{\alpha}\hat{Q}_{\beta}\right) S^{\alpha\beta}\left(\boldsymbol{Q},\omega\right)$$
$$S^{\alpha\beta}\left(\boldsymbol{Q},\omega\right) = \sum_{r_{i}} \sum_{r_{j}} \exp\left(i\boldsymbol{Q}\cdot\left(\boldsymbol{r}_{i}-\boldsymbol{r}_{j}\right)\right) \int_{-\infty}^{\infty} \left\langle S_{r_{i}}^{\alpha}\left(0\right) S_{r_{j}}^{\beta}\left(t\right)\right\rangle \exp\left(i\omega t\right) dt$$

For elastic neutron scattering it becomes

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\left(\gamma r_{0}\right)^{2}}{2\pi\hbar} \frac{k_{f}}{k_{i}} \left[F\left(\boldsymbol{Q}\right)\right]^{2} \exp\left(-2W\right) \sum_{\alpha,\beta} \left(\delta_{\alpha,\beta} - \hat{Q}_{\alpha}\hat{Q}_{\beta}\right) \sum_{r_{i}} \sum_{r_{j}} \exp\left(i\boldsymbol{Q}\cdot\left(\boldsymbol{r}_{i}-\boldsymbol{r}_{j}\right)\right) \left\langle S_{r_{i}}^{\alpha}S_{r_{j}}^{\beta}\right\rangle$$

- F(Q) Magnetic form factor which reduces intensity with increasing wavevector
- exp<-2W> Debye-Waller factor which reduces intensity with increasing temperature

 $\left(\delta_{\alpha,\beta} - \hat{Q}_{\alpha}\hat{Q}_{\beta}\right)$ polarisation factor which ensures only components of spin perpendicular to Q are observed

 $\left\langle S_{r_i}^{\alpha}(0)S_{r_j}^{\beta}(t)\right\rangle$ is the spin-spin correlation function which describes how two spins separated in distance and time a related

Distinguishing Phonons and Magnons with Neutrons

Wavevector-dependence

- Structural excitations have high intensity at large |Q| and when Q is parallel to the mode of vibration
- Magnetic excitations have high intensity at low |Q| and when Q is perpendicular to the magnetic moment direction

Temperature dependence

- Structural excitations become stronger as temperature increases
- Magnetic excitations become weaker as temperature increases

Inelastic neutron scattering

- both the initial and final neutron energy

Triple-axis spectrometer

The initial and final neutron energies can be selected or measured using monochromator and analyser crystals where the wavelength of the neutrons is determined by the scattering angle.

Time-of-flight Spectrometer.

The initial and final energies are selected or measured using the time it takes the neutron to travel through spectrometer to the detector from this the velocity and hence kinetic energy are deduced.

The Triple Axis Spectrometer - Layout



The Triple Axis Spectrometer – Monochromator Analyser

The monochromator is a crystalline material and selects a single wavelength from the white neutron beam of the reactor/spallation source by Bragg scattering where the scattering angle is chosen to select λ . The analyser measures the final neutron energy



Vertically focusing monochromator



from graphite, Copper, Germanium, blades can be focused

Horizontally focusing monochromator



The Triple Axis Spectrometer – V2/FLEX, HZB



The Triple Axis Spectrometer – Measurements

Keep wavevector transfer constant and, scan energy transfer.



Keep energy transfer constant and, scan wavevector transfer.





Triple Axis Spectrometer – Pros and Cons

Advantages

- Can focus all intensity on a specific point in reciprocal space
- Can make measurements along high-symmetry directions
- Can use focusing and other 'tricks' to improve the signal/noise ratio
- Can use polarisation analysis to separate magnetic and phonon signals

Disadvantages

- Technique is slow and requires some expert knowledge
- Use of monochromator and analyser crystals gives rise to possible higher-order effects that are known as "spurions"
- With measurements restricted to high-symmetry directions it is possible that something important might be missed

Time of Flight Spectrometer – Layout of V3/NEAT

Time and distance are used to calculate the initial and final neutron velocity and therefore energy. This is achieved by cutting the incident beam into pulses to give an initial time and incident energy







Time of Flight Spectrometer - Choppers

The neutron beam is cut into pulses of neutrons using disk choppers.

 I^{st} chopper spins letts neutrons through once per revolution and sets initial time $t_{\rm 0}$

2nd chopper spins at the same rate and opens at a specific time later. The phase is chosen to select neutrons of a specific velocity and energy.



After scattering at the sample the detector again measures time as well as number of neutrons, thus the velocity and energy of the scattered neutrons is known.

The Time of Flight Spectrometer - Choppers







• First chopper sets the initial time.

- Second chopper sets the initial energy
- Detectors measure final time and energy.

B. Lake; Tartu, Sept 2017

Time of Flight Spectrometer – Detectors



Time of Flight Spectrometer – Measuring



- Every detector trances a different path in E and Q transfer
- A large dataset is obtained from all detectors containing intensity as a function of three dimensional wavevector and energy

Time of Flight Spectrometer – Pros and Cons

Advantages

- It is possible to simultaneously measure a large region of energy and wavevector space and get an overview of the excitations
- This allows unexpected phenomena to be observed
- It does not have the same problem of second order scattering as the triple axis spectrometer

Disadvantages

- Time-of-flight instrument have low neutron flux for an specific wavevector and energy but the ESS will be different
- It is difficult to do polarised neutron scattering



Example 1 Zero Dimensional Quantum Magnets

0-Dimensions - Spin-1/2, Dimer Antiferromagnets





Zeeman Splitting in Field B Triplet, s=1. Singlet, s=0. B. Lake; Tartu, Sept 2017

Bose Einstein Condensation



Properties:

- Singlet ground state.
- Gapped 1-magnon
- 2-magnon continuum
- Bound modes.
- Bose Einstein condensation.

$Sr_3Cr_2O_8$ – Spin-1/2, Dimer AF

$Sr_3Cr_2O_8 \rightarrow Cr^{5+}, Spin-1/2.$

Space group - R-3m





E_{gap}=3.4meV E_{upper}=7.10meV

Dimer coupling is bilayer J_0

Sr₃Cr₂O₈ is 3D network of dimers

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$$E_{midband} \sim J_0 = 5.5 meV$$

 $E_{bandwidth} \sim J' = 3.7 meV$

Powder inelastic neutron scattering

D.L. Quintero-Castro, et al Phy. Rev. B. 81, 014415 (2010)





Single Crystal Inelastic Neutron Scattering



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D.L. Quintero-Castro, et al Phy. Rev. B. 81, 014415 (2010)

Fitting to a Random Phase Approximation

$\begin{array}{c} 7.0 & \Gamma & M & \Gamma & M' \\ 6.5 & 0 & 0 & 0 \\$

Extracted Dispersions

Random Phase Approximation

M. Kofu et al Phys. Rev. Lett. 102 037206 (2009)

$$\hbar\omega \simeq \sqrt{J_0^2 + J_0\gamma(\mathbf{Q})} \qquad \gamma(\mathbf{Q}) = \sum_i J(\mathbf{R}_i)e^{-i\mathbf{Q}\cdot\mathbf{R}_i}$$

	Constants	$\rm Sr_3 Cr_2 O_8$
	J_0	5.551(9)
	J_1'	-0.04(1)
	J_1''	0.24(1)
	J_{1}'''	0.25(1)
J_2	$J_2' - J_3'$	0.751(9)
la T	$J_{2}^{\prime \prime }-J_{3}^{\prime \prime }$	-0.543(9)
	$J_{2}^{\prime \prime \prime \prime} - J_{3}^{\prime \prime \prime \prime}$	-0.120(9)
>>	J'_4	0.10(2)
J ₁	J_4''	-0.05(1)
	$J_A^{\prime\prime\prime\prime}$	0.04(1)
,d ₀	J' =	J' = 3.6(1)
	J'/J_0	$J'/J_0 = 0.6455$

Simulation and Data



D. L. Quintero-Castro, B. Lake, E.M. Wheeler Phy. Rev. B. 81, 014415 (2010)

Simulation of the TOF data with the fitted values interactions



Example 2 One Dimensional Quantum Magnets



1D, S-1/2, Heisenberg, Antiferromagnet

$H = J \sum_{i} \vec{S}_{i} \cdot \vec{S}_{i+1}$

Bethe Ansatz

- Ground state has no long-range Néel order.
- Ground state consists of 50% spin-flip states
- All combinations must be considered.
- Little physical insight into the quasi-particles.



Hans Bethe Bethe Ansatz (1931)

The Bethe Ansatz has been a long standing problem of theoretical condensed matter

Spinons Excitations

Fadeev and Taktajan (1981)

The fundamental excitations are spinons not magnons.









Spinons

- Fractional spin-½ particles
- created in pairs
- spinon-pair continuum

Solution of Bethe Ansatz

Several approximate theories have since been postulated for the spinon continuum of the spin-1/2 Heisenberg chain

- Müller Ansatz
- Luttinger Liquid Quantum Critical point

In 2006 J.-S. Caux and J.-M. Maillet solved the 1D, spin-1/2, Heisenberg, antiferromagnet, 75 years after the Bethe Ansatz was proposed.

all-spinon

J.-S. Caux, R. Hagemans, J. M. Maillet (2006)





1D S-1/2 Heisenberg Antiferromagnetic - KCuF₃

Cu²⁺ ions S=1/2 Antiferromagnetic chains, $J_{//}$ = -34 meV Weak interchain coupling, $J_{\perp}/J_{//} \sim 0.02$ Antiferromagnetic order $T_N \sim 39K$ Only 50% of each spin is ordered

$$\hat{H} = J_{\parallel} \sum_{r} \vec{S}_{r,l} \cdot \vec{S}_{r+1,l} + J_{\perp} \sum_{l,\delta} \vec{S}_{r,l} \cdot \vec{S}_{r,l+\delta}$$









KCuF₃ compared to Bethe Ansatz, 2 and 4 spinons





Example 3 Two Dimensional Quantum Magnets

2-Dimensional Antiferromagnet - Square Lattice



Rb₂MnF₄ 2-Dimensional Spin-5/2 Heisenberg Antiferromagnet



T Huberman et al J. Stat. Mech. (2008) P05017



2-Dimensional Antiferromagnet - Triangular Lattice

Triangular Lattice

Ground state - long range order



Excitations A Mezio, et al New Journal of Physics (2012)



CuCrO₂ S-3/2, triangular lattice

M Frontzek et al Phys. Rev. B (2011)



Alpha-Ca₂CrO₄ S-3/2, triangular lattice

S Toth et al Phys. Rev. B (2011)



Ideal S-1/2, triangular antiferromagnet, Ba₃CoSb₂O₉ H. Tanaka et al

2-Dimensional Antiferromagnet - Kagome Lattice

Kagome Lattice





S-1/2 no order diffuse excitations



e.g. Herbertsmithite *T.-H. Han Nature 492, 406 (2012)*



 $J_{2} = 0$

Pseudo-Fermion (b) Functional Renormalisation Group *R. Suttner, et al Phys. Rev. B* (2014)

Group (4) $\chi(k)^{1.0}$ 0.50.0 $-\frac{6\pi}{3} - \frac{4\pi}{3} - \frac{2\pi}{3} k_x - \frac{2\pi}{3} - \frac{4\pi}{3} - \frac{4\pi}{3} - \frac{4\pi}{\sqrt{3}}$

S-5/2 Long-range order B. L Spin-wave excitation

Ca₁₀Cr₇O₂₈ - Crystal structure



space group R3c

D. Gyepesova, Acta Cryst. C69, 111 (2013)

B. Lake; Tartu, Sept 2017

- Cr^{5+} spin = $\frac{1}{2}$ ions (1 electron in 3d-shell)
- 7 different exchange path in structure
- No long-range magnetic order

Kagome bilayer model

- *a-b* plane shows distorted kagome bilayers
- large blue and small green triangles alternate within each layer, and between layers



Inelastic Neutron Scattering – Zero Field

Powder; TOFTOF, FRM2; T=0.43K



Two Bands of excitations



[H,K,0]; MACS, T=0.09K





C. Balz, B.Lake, J. Reuther et al Nature Phys. 12, 942 (2016)

Broad diffuse scattering

Inelastic neutron scattering – Zero field, single crystal



Broad Diffuse Scattering no spin-waves, spinons?



B. Lake; Tartu, Sept 2017

C. Balz, B. Lake, J. Reuther, Nature Physics 12, 942–949 (2016)

	Ca ₁₀ Cr	₇ O ₂₈ - Ma	agne	tic model - Exchange couplings
Exchange	Cr-Cr distance	Coupling constant	Туре	$\mathcal{H} = J_{ij} \sum_{ii} S_i \cdot S_j$
	d [Å]	J [meV]		1 EM introbilovor
JO	3.883	-0.0794	FM	
J21	5.033	-0.7615	FM	EM triangles
J22	5.095	-0.2696	FM	J I WI WINNINGIES
J31	5.697	0.0876	AFM	
J32	5.750	0.1072	AFM	J AFIVI IIIaligies
ΣJ		-0.9158		

Pseudo-Fermion Functional Renormalisation Group

Using the Hamiltonian extracted from INS

- \Rightarrow Susceptibility shows no long-range order
- \Rightarrow Diffuse magnetic scattering



Non-ordered ground state, diffuse spinon scattering. Reveals highly robust spin liquid state





Example 4 Three Dimensional Quantum Magnets

Frustrated 3-Dimensions Magnets – Pyrochlore Lattice

Pyrochlore Lattice – corner-sharing tetrahedra





Interconnected chains Antiferromagnetic J 3D frustration

MgV₂O₄, V has spin-1



B. Lake; Tart Constant Energy E=8meV, IN20 with Flatcone reveals broad diffuse scattering very different from spin-wave excitations

3-Dimensions - Pyrochlore Magnets

Spin Ice

Ferromagnetic interactionsStrong Ising anisotropyIce rules 2 in, 2 out







Ground state - topological order

Excitations - monopoles



Conventional Magnets

Long-range magnetic order and spin-wave excitations

Origins of Quantum magnetism

low spin values, antiferromagnetic, low-dimensional, frustration, spin liquids

Neutron scattering for quantum magnets Inelastic neutron scattering Triple Axis spectrometer, time-of-flight spectrometer

Examples of frustrated magnets 0-dimensional magnets e.g. dimer magnets 1.Dimensional magnets e.g. the spin-1/2 chain 2-Dimensional magnets e.g. Square, triangular, kagome, lattice 3-Dimensional magnets e.g. pyrochlore, spin ice and water ice

Beyond long-range magnetic order and spin-wave theory there are many unusual quantum states to be explored